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Bianchi Type-III cosmological model with cloud string bulk viscosity

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Abstract

In this paper, we have discussed on Bianchi type-III space-time with bulk viscosity in the context of Einstein theory. To study the model, we have considered some realistic plausible conditions and obtained a physically acceptable model that indicates an accelerating expanding Universe. The physical and geometrical significance of the model have been discussed.

Keywords: Bianchi type-III, Einstein theory, cosmological model, bulk viscosity, cloud string.

Introduction

Our universe is full of mysterious heavenly bodies. The heavenly bodies are still unable to uncover after a huge number of findings with modern technology, only a little able to know regarding our universe. To study the universe and its fates, researchers are developing different types of cosmological models. The cosmological models are being developed and studied using different theories such as Einstein theory, alternative theories, Weyl theory, Lyra geometry, $f(R,T)$ theory etc. Many authors studied the universe using verities of metric space-time using string in different theories. RK Dabgar *et al.* investigated on five-dimensional Bianchi type-III string cosmological model using both power-law model and exponential model that provide insights into the behaviour of the universe's evolution and expansion under the influence of dark energy [1]. R. K. Tiwari *et al.* analysed Bianchi type III string cosmological model in f (R, T) modified theory of gravity by assuming $f\ (R,T) = R + 2f(T)$ which concluded with a note that the universe is in accelerating phase of expansion [2]. Jiten B *et al.* studied Bianchi type-III cosmological model with a cloud string with particles connected to them in Lyra geometry and concluded that the present model starts at $t = 0$ with zero volume and as time progresses it expands with accelerated rate and the model shows that the present universe is particle dominated [3]. Kandelkar et. al examined Bianchi type-III string cosmological model in the presence of magnetic field. To get determinate solutions, the Einstein's field equations have been solved for two cases (i) Reddy string and (ii) Nambu string [4]. Swati *et al.* studied the behaviour of cosmological constant Λ in Bianchi Type III string cosmological model for dust fluid [5]. R. D. Upadhayaa *et al.* studied Some Bianchi type III cosmological models with magnetic field for massive string are investigated [6]. J.K singh *et al.* studied Spatially homogeneous and totally anisotropic Bianchi type-III cosmological models in the theory based on Lyra's geometry in Gauss normal gauge in the presence of an attractive massive scalar field, using the special law of variation for Hubble's parameter and concluded that one of the universe models approaches to isotropy through the evolution of the universe, in some special cases [7]. S.D deo *et al.* studied Bianchi type -III Cosmological model filled with a Magnetized Cosmic Strings is investigated in general relativity [8]. KS Adhav *et al.* studied on Bianchi type-III cosmological model in General Relativity with linear equation of state (EoS) i.e $p = \alpha\rho - \beta$, where α and β are constants [9]. Priyanka Kumari *et al.* studied on anisotropic Bianchi type-III cosmological model in the presence of a bulk viscous fluid within the framework of Lyra geometry with time-dependent displacement vector [10].

Cosmic strings are the main source in rising density perturbations that are responsible for galaxy formation in the early universe [11-17]. Also, the bulk viscosity mechanism in cosmology describes the present scenario of high entropy and accelerated expansion of the universe. At an early stage, the coupling of neutrinos disappears, and matter distribution in the universe act as a bulk viscous fluid. Cosmological models with bulk viscosity are important since bulk viscosity has a greater role in getting accelerated expansion of the universe popularly known as inflationary. The modern findings in cosmology tells us that the universe is expanding and accelerating [18-20]. Observations from type-Ia Supernova [21-24], CMB radiation [25, 26] and LSS [27] are the evidence that the current universe is having an accelerated expansion, rather than slowing down as predicted by the big bang theory [28]. Scientists are trying to solve this accelerating universe by assuming various probabilities. But till today, they could not arrive at a satisfactory conclusion on such strange behaviour of the universe. The behaviour of late-time acceleration of the universe cannot be satisfactorily described by the general theory of relativity, although it is considered as the most successful theory in describing the early evolution of the universe. Cosmologists have arrived at two possible approaches to answer this cosmic accelerating expansion. One of such approaches is to introduce dark energy which dominates the universe and has associated with negative pressure. The second consideration is to modify Einstein's general theory of relativity. The Bianchi models explain the correct matter distribution and helps in understanding the beginning phases of development of the universe. Several authors studied spatially homogeneous anisotropic Bianchi models to get the relativistic picture of early universe. Moreover, many authors have examined diverse Bianchi models with normal impeccable liquids [29-34].

Inspired from the above researchers we have considered a Bianchi type-III cosmological model with cloud string in the presence of Bulk viscosity. The plan of this research work is as follows: In Sect. 2, we present the Metric and the field equations for the model assumed. In sect. 3 we perform the solution of the model. In sect. 4 we present the geometrical and physical significances of the model. Finally, in sect. 5 we discuss on our results.

The metric and the field equations

Bianchi type III metric in the form

$$ds^2 = A^2\,dx^2 + B^2 e^{-2\propto x}\,dy^2 + C^2 dz^2 - dt^2 \tag{1}$$

Where A, B and C are functions of cosmic time t only.

Using the variation AL principle, we derive the fundamental equations for the gravitational field

$$S(g,\rho) = \int \mathcal{L}\sqrt{g}\,d\Omega \tag{2}$$

With $\mathcal{L} = \mathcal{L}_{grav} + \mathcal{L}_{vf}$

Where ρ is energy density of the viscous fluid, Ω is an arbitrary integration region.

The energy momentum tensor for a cloud string dust with a bulk viscous fluid of string is given by,

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta\left(g_{ij} + u_i u_j\right) \tag{3}$$

Where u_i and x_i satisfying the conditions

$$u_i u^i = -x_i x^i = 1,\ u_i x^i = 0 \tag{4}$$

Where ρ is the proper energy density for a cloud string with particles attached to them, λ is a string tension density, u_i is the four-velocity vector of the particle, x^i is the unit space-like vector representing the direction of the string. In a comoving co-ordinate system, we have

$$u^i = (0,0,0,1),\ x^i = \left(0,0,\frac{1}{c},0\right) \tag{5}$$

The particle density of the configuration is denoted by ρ_p,then we have

$$\rho = \rho_p + \lambda \tag{6}$$

The Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\,R g_{\mu\nu} \tag{7}$$

The Einstein field equation (in gravitational units C=1,8πG=1)

$$R_{ij} - \frac{1}{2}\,R g_{ij} = T_{ij} \tag{8}$$

Einstein field equations are

$$\frac{\dot B\dot C}{BC} + \frac{\ddot B}{B} + \frac{\ddot C}{C} = \xi\theta \tag{9}$$

$$\frac{\dot{A}\dot{C}}{AC}+\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}=\xi\theta \tag{10}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}-\frac{\alpha^2}{A^2}=\xi\theta+\lambda \tag{11}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{\alpha^2}{A^2}=\rho \tag{12}$$

$$\alpha\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0 \tag{13}$$

The particle density ρ_p, Expansion scalar Θ, shear scalar σ^2 is given by

$$\rho_p=\frac{\ddot{C}}{C}-\frac{\ddot{A}}{A}+3\frac{\dot{A}}{A}\frac{\dot{C}}{C} \tag{14}$$

$$\Theta=2\frac{\dot{A}}{A}+\frac{\dot{C}}{C} \tag{15}$$

$$\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij}=\frac{1}{3}\left(\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right)^2 \tag{16}$$

Solution of the field equation

We assume that the expansion Θ in the model is proportional to the shear scalar σ. this condition leads to.

$$B=C^n, \tag{17}$$

Where n is the constant.

To obtain the determinate model of the universe, we assume that the co-efficient of the bulk viscosity is inversely proportional to the expansion factor θ. This condition leads to

$$\xi\theta=K, \tag{18}$$

where K is the proportionality constant.

Equation (13) leads to

$$A=mB, \tag{19}$$

where m is the integrating constant.

Putting these values in the field equation we get.

$$\ddot{C}+\left(\frac{n^2}{n+1}\right)\frac{\dot{C}^2}{C}=\frac{KC}{n+1} \tag{20}$$

Let $\dot{C}=f(C)$

Hence the equation (20) leads to.

$$\frac{d}{dC}(f^2)+\frac{n^2}{n+1}\frac{f^2}{C}=\frac{KC}{n+1} \tag{22}$$

After integration

$$f^2=\frac{KC^2}{n^2+n+1}+\frac{L}{C^{\frac{-2n^2}{n+1}}} \tag{23}$$

Where L is integrating constant.

Thus, the given matric reduces to the form.

$$ds^2=C^{2n}\,dx^2+C^{2n}e^{-2\propto x}\,dy^2+C^2dz^2-\left(\frac{dt}{dC}\right)dC^2 \tag{24}$$

After the transformation, the above metric reduces

$$ds^2=T^{2n}\left(dX^2+e^{-2\propto X}\,dY^2\right)+T^2dZ^2-\frac{dT^2}{\frac{KT^2}{n^2+n+1}+LT^{\frac{-2n^2}{n+1}}} \tag{25}$$

Some physical and geometrical features

The physical significance of ρ, θ,σ, λ, ρ_p are given by.

$$\rho = \frac{(n^2+2n)K}{n^2+n+1} + \frac{(n^2+2n)L}{T^{2\left(\frac{n^2+n+1}{n+1}\right)}} - \frac{\alpha^2}{T^{2n}} \tag{26}$$

$$\theta = \frac{2n+1}{T}\left(\frac{KT^2}{n^2+n+1} + LT^{\frac{-2n^2}{n+1}}\right)^{\frac{1}{2}} \tag{27}$$

$$\sigma^2 = \frac{(n-1)^2}{3}\left(\frac{K}{n^2+n+1} + \frac{L}{T^{2\left(\frac{n^2+n+1}{n+1}\right)}}\right) \tag{28}$$

$$\lambda = \frac{(2n^2+n)K}{n^2+n+1} + \frac{n(n-1)L}{(n+1)T^{2\left(\frac{n^2+n+1}{n+1}\right)}} - \frac{\alpha^2}{T^{2n}} - K \tag{29}$$

$$\rho_p = \rho - \lambda = \frac{K(2n+1)}{n^2+n+1} + \frac{n(n^2+2n+3)L}{(n+1)T^{\frac{2n^2+2n+3}{n+1}}} \tag{30}$$

The model is being studied for different values of n and the values of arbitrary constants are choosen as such the model describe a realistic universe.

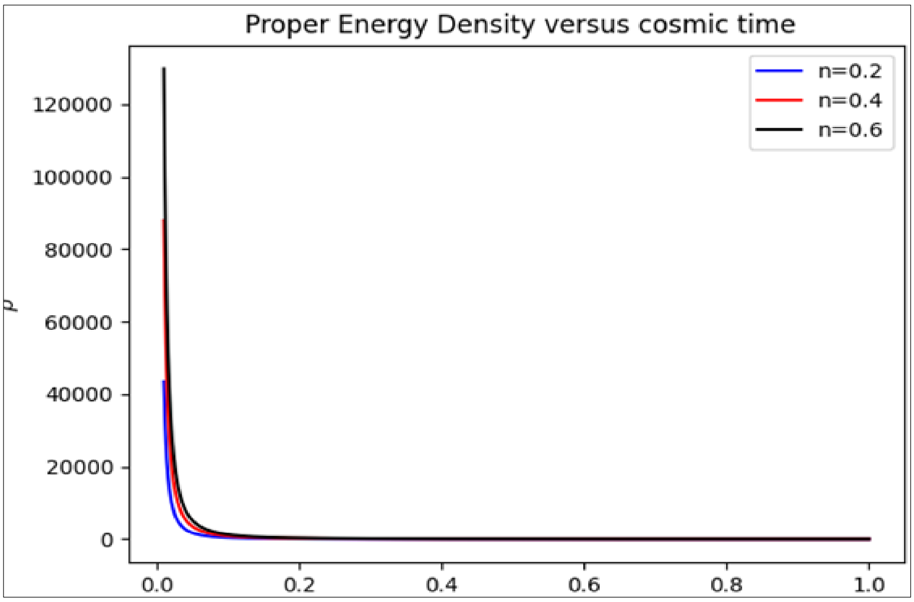

Fig 1: Plot of proper energy density versus cosmic time for K=1.05, $\alpha = 0.56$, $L = 10.2$

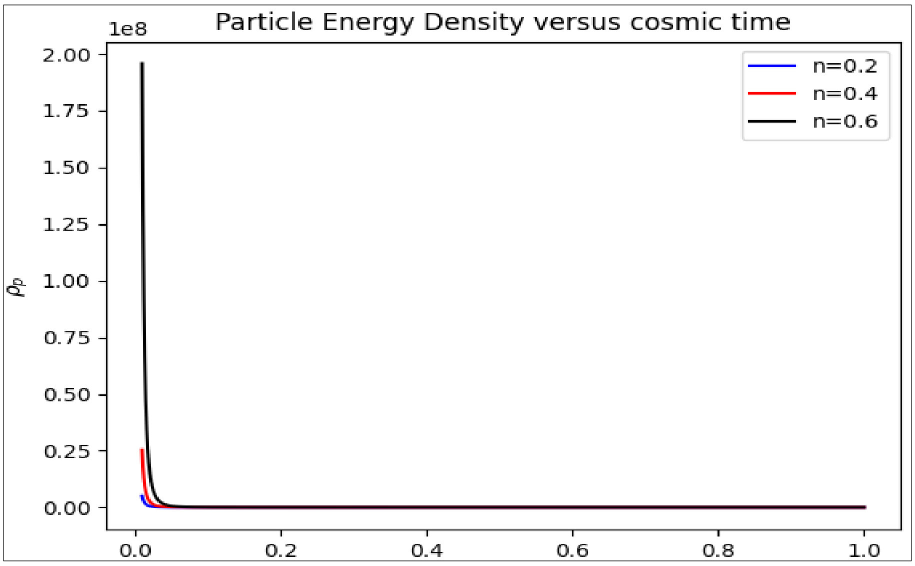

Fig 2: Plot of particle energy density versus cosmic time for K=1.05, $\alpha = 0.56$, $L = 10.2$

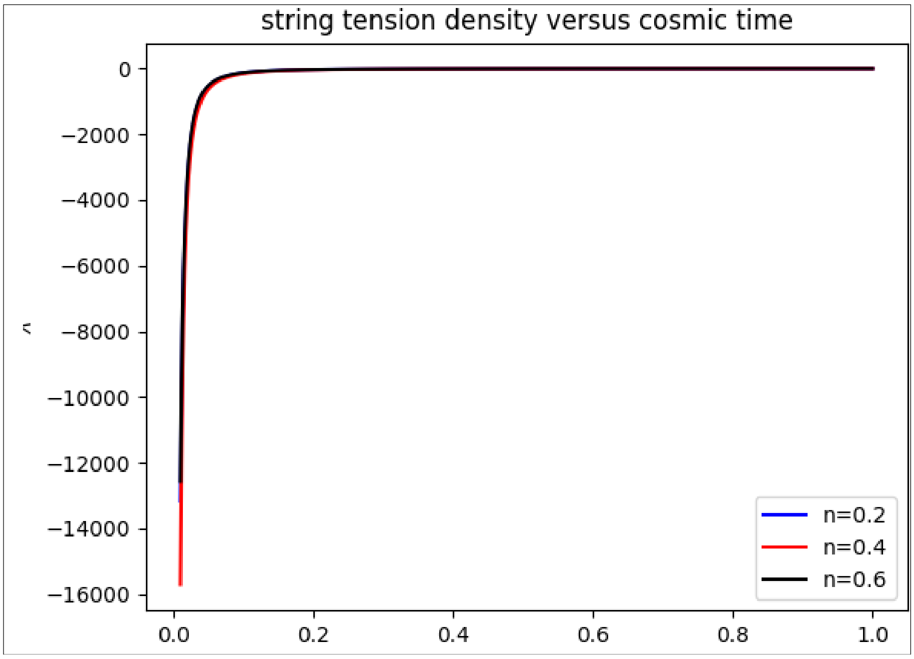

Fig 3: Plot of string energy density versus cosmic time for K=1.05, $\alpha = 0.56$, $L = 10.2$

The above figures describes the accelerating expansion of the universe which matches with the result [18-28].

Conclusion

In this paper we have studied on Bianchi type-III cosmological model with the help of Cloud string and Bulk viscosity in the context of Einstein theory. The model being studied using plausible realistic conditions. The obtained model is isotropic in nature. The proper energy density is diverging in nature initially and later it approaches to zero and it is positive throughout the entire life span. The particle energy density is also obtained as diverging in nature initially which indicates the Big-Bang and later vanishes. Lastly the string energy density is diverging initially on negative side and later it increases and approaches to zero. The results obtained is acceptable as per the recent observational views.

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