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On the probability distribution of surviving children

AK Tiwari and Shivam Mishra

Abstract
Most of the decisions concerning the future fertility behaviour are prejudiced by the number of surviving children instead of number of children ever born. Although, for studying the variation in the number of births occur to a female in a given period, various probability models are developed under the varying set of assumptions, yet very few studies are found to derive the distribution of number of surviving children out of the total births in a specified period. Thus it appears necessarily to attempt for finding the distribution of number of births as well as the probability distribution of number of surviving children. The specific period in which the study of conceptions and surviving children is carried out, is divided into two process i.e. equilibrium birth process and free process. Yadava and Tiwari (2006), derived the distribution of surviving children using some laws of probability. Hence, in this present paper, probability distribution of surviving children out of the total births, and average number of survived children in the free process is calculated by using same laws of probability and set of assumptions. The distribution of number of births and surviving children in the equilibrium birth process and free process is compared by considering the current mortality level 20 from West model life table.

Keywords: Surviving children, equilibrium birth process, free process, probability distribution

1. Introduction

During the last decades, a large number of models have been proposed to explain the number of births. Dandekar (1955) [1] used modified forms of Binomial and Poisson Distributions to explain the number of children born to a cohort of females in fixed time with the major assumption that all females are susceptible to the risk of the conception at the beginning and it leads to birth. Brass (1958) [2] used negative binomial distribution to explain the distribution of births during a specified period. Singh (1964, 68) [3,4] modified the model of Dandekar assuming that some proportions of females are susceptible to the risk of conception throughout the observation period and the remaining are incapable to get conception during this period. Apart from other assumptions, Perrin and Sheps (1964) [5], Singh and Bhattacharya (1970, 71) [6,7], Singh *et al*. (1974) [8] have proposed models by assuming that the females are exposed to the risk of conception at the beginning of the interval $(0, T)$.

Dandekar (1955) [1], Singh and Yadava (1977) [9], Singh and Singh (1981) [10], etc. have also proposed the models to study the number of births in a given period $(T_0, T_0 + T)$ of length T where T_0 is a distant point since marriage. Singh and Yadava (1977) [9] have named this situation as ‘Equilibrium Birth Process’. The main property of this process is that the births are uniformly distributed over the time but it is not true for the interval $(0, T)$. To investigate this claim, Yadava and Srivastava (1993) [11] have tried to find out the distribution of births over the time for a female giving a specified number of births in a given period T in the equilibrium birth process. They derived the distribution of births by dividing the whole interval into segments of length h, where h is the non-susceptibility period (sum of the gestation period and postpartum amenorrhea). This probability distribution is quite useful in finding out the distribution of the number of surviving children out of the births in a given period. The knowledge of the probability distribution of surviving children may be used on the future fertility behavior of a couple.

Yadava and Tiwari (2007) [12] derived similar distribution of conceptions over the time in the interval $(0, T)$ for a female having a specified number of conceptions in the given period. They have also derived probabilities as when the lengths of segments are one year rather than h.

To make the difference between the equilibrium birth process and the interval of type $(0, T)$, Yadava and Tiwari (2007) [12] have termed this situation i.e. interval of type $(0, T)$ as 'Free Process'.

1.1 Pattern of the distribution of births over the time in equilibrium birth process and free process

Yadava and Srivastava (1993) [11] investigated the pattern of the distribution of births over the time in the equilibrium birth process. The probability that there are X_1 conceptions in the first interval of length h, X_2 in the second interval of length h, X_3 in the third interval of length h, and X_n in nth (last) interval of h, is denoted by $P_{X_1X_2X_3\cdots X_n}$. For the numerical illustration, they derived the distribution of births for $T = 4h$ and $T = 5h$ (given in Yadava and Tiwari, 2010) [13]. The value of h and λ is taken as 1.5 years and 0.5 respectively. After review of the values of the probabilities of the type $P_{X_1X_2X_3\cdots X_n}$ (Yadava and Tiwari, 2007) [12], they reveal that the probabilities are symmetric $P_{10} = P_{01}$, $P_{100} = P_{001}$, $P_{110} = P_{011}$, $P_{0001} = P_{1000}$, $P_{1100} = P_{0011}$, $P_{1110} = P_{0111}$, $P_{00001} = P_{10000}$, $P_{11000} = P_{00011}$, $P_{11100} = P_{00111}$, etc.

The mathematical justification for observed symmetries in the equilibrium birth process is given in Yadava and Tiwari (2010) [13]. Next, Yadava and Srivastava (1993) [11] observed that r births occurred in the equilibrium birth interval $(T_0, T_0 + T)$ are not uniformly distributed over the time interval $(T_0, T_0 + T)$.

Yadava and Tiwari (2007) [12] described the procedure for finding the distribution of births over the time in the interval $(0, T)$ and the same type of probabilities $P^*_{X_1X_2X_3\cdots X_n}$ are obtained. They have also calculated similar probabilities when the length of segments is one year in place of h

The major difference between the equilibrium birth process and the free process is that in the free process at the beginning of the segment, the female is exposed to the risk of conception because in many cases this starting point is taken as marriage. While in the equilibrium birth process there is a possibility also that females may remain non-susceptible at the beginning of the segment. It has been observed that probabilities $P^*_{X_1X_2X_3\cdots X_n}$ are not symmetric in the free process i.e. $P^*_{X_1X_2X_3\cdots X_n} \neq P^*_{X_nX_{n-1}X_{n-2}\cdots X_1}$. This happens may be due to the property (uniform distribution of births over the time) does not hold well in the free process. Another type of equality was found. Once, we analyzed and compared the estimated probabilities for equilibrium birth process and free process in case of $T = 4h$ and $T = 5h$ (Yadava and Tiwari, 2007) [12], we found that $P^*_{1000} > P_{1000}$, $P^*_{1010} > P_{1010}$, $P^*_{1100} > P_{1100}$, $P^*_{1110} > P_{1110}$, $P^*_{1101} > P_{1101}$, etc. and $P^*_{00000} < P_{00000}$, $P^*_{00001} < P_{00001}$, $P^*_{11000} > P_{11000}$, $P^*_{10010} > P_{10010}$, etc. It indicates that the probability distribution of births over the time, for a female having a specified number of children in a given period in equilibrium birth process and in the free process is different which means the pattern of the birth over the time in both processes are not the same. Even different probabilities were found when the length of the segments is one year.

These probabilities are very much useful in deriving the distribution of the number of surviving children out of the total births in a given period. Yadava and Tiwari (2006) [14] derive the probability distribution of the number of surviving children out of the total births in the equilibrium birth process. To derive the probability distribution of surviving children, they used basic laws of probability and making certain assumptions regarding survival probabilities for the birth occurring in a particular segment. Since the pattern of the births over the time in equilibrium birth process and the free process is different, so it is logical to derive the probability distribution of surviving children in the free process using the same laws of probabilities and assumptions on survival probabilities used by Yadava and Tiwari (2006) [14].

In the present paper, authors have derived the probability distribution of surviving children in the free process and then compared the probability distribution of surviving children in the equilibrium birth process and in the free process.

2. Materials and Methods

In the present study, the probabilities $P^*_{X_1X_2X_3\cdots X_n}$ are taken from Yadava and Tiwari (2007) [12] and the probability of survival can be obtained from the model life table. According to SRS-based life table 2010-14, the life expectancy at birth in India is 67.9 years. Hence, the probability distribution of surviving children in the equilibrium birth process as well as in the free process is calculated by taking the mortality level 20 in the West model life table.

2.1 Procedure to derive probability distribution of surviving children

To derive the distribution of the number of surviving children out of the total births at the end of the observational period in equilibrium birth process, Yadava and Tiwari (2006) [14] used the following assumptions and information:

1. The births occur at the midpoint of the segment.
2. The timing of the midpoint of the segment and the survey date is known, then the time length for survival can be identified.
3. To calculate the survival probability from births to this time length, the life table for the given population can be used. If the life table is not known then an appropriate model life table may be considered with the same mortality level for a given population.
4. Once the survival probabilities of birth occurring in different segments are known then laws of probabilities can be used to find the probability distribution of survivors out of the total births occur in the given period.

These assumptions and information can be used to compute the number of surviving children in the free process with an understanding that the number of conceptions in the interval $(0, T)$ will correspond to the number of births in $(g, g + T)$ where g is the gestation period.

2.2 Average Number of Surviving Children for $T = 4h$ in Free Process

Here we have considered $T = 4h$ in the free process. If $h = 1.5$ years, hence the observational period is 6 years and there will be 16 probabilities that can be obtained from Yadava and Tiwari (2007) [12].

The average number of surviving children out of total birth in $4h$ will be.

$$= [0 \times Pr(zero\ survivors\ in\ the\ whole\ period)] + [1 \times Pr(one\ survivor)] + [2 \times Pr(two\ survivors)] + [3 \times Pr(three\ survivors)] + [4 \times Pr(four\ survivors)] \quad (1)$$

2.2.1 Probability of zero survivors

The union of the following mutually exclusive events will explain the probability of zero survivors in the whole period i.e. $4h$ (6 years).

- $\boldsymbol{E_0^0}$: There is no birth in the given period.

- $\boldsymbol{E_1^0}$: There is only one birth in the given period of 6 years and that child does not survive till the exposure period for survival.
- $\boldsymbol{E_2^0}$: There are exactly two births in the given period and none of these two children survives till the survey date.
- $\boldsymbol{E_3^0}$: There are exactly three births and none of these three children survives till the survey date.
- $\boldsymbol{E_4^0}$: There are exactly four births and none of them survives till the survey date.

2.2.2 Probability of one survivor

The event is the union of four mutually exclusive events, which are given as follows:

- $\boldsymbol{E_1^1}$: There is exactly one birth in the whole period and the child survives till the survey date.
- $\boldsymbol{E_2^1}$: There are exactly two births in the period and one of them survives till the survey date.
- $\boldsymbol{E_3^1}$: There are exactly three births in the period and one of them survives till the survey date.
- $\boldsymbol{E_4^1}$: There are exactly four births in the period and one of them survives till the survey date.

2.2.3 Probability of two survivors

The event is the union of three mutually exclusive events, which are given as follows:

- $\boldsymbol{E_2^2}$: There are exactly two births in the whole period and both children survive till the survey date.
- $\boldsymbol{E_3^2}$: There are exactly three births in the period and only two of them survive till the survey date.
- $\boldsymbol{E_4^2}$: There are exactly four births in the period and only two of them survive till the survey date.

2.2.4 Probability of three survivors

The event is the union of two mutually exclusive events, which are given as follows:

- $\boldsymbol{E_3^3}$: There are exactly three births in the whole period and all three children survive till the survey date.
- $\boldsymbol{E_4^3}$: There are exactly four births in the period and any three of them survive till the survey date.

2.2.5 Probability of four survivors

Now consider the probability of four survivors, the event will be described as one birth occurs in the first segment, one birth occurs in the second segment, one birth occurs in the third segment, and one birth occurs in the fourth segment; and all of them survive up to the survey date. Then the probability can be estimated as

$$
\begin{aligned}
Pr[X=4] &= P^*_{1111} \\
&\times [\{Probability\ of\ survival\ from\ birth\ to\ period\ (4h-0.5h)\} \\
&\times \{Probability\ of\ survival\ from\ birth\ to\ period\ (4h-1.5h)\} \\
&\times \{Probability\ of\ survival\ from\ birth\ to\ period\ (4h-2.5h)\} \\
&\times \{Probability\ of\ survival\ from\ birth\ to\ period\ (4h-3.5h)\}] \\
Pr[X=4] &= 0.00730 \\
&\times [\{0.947490\} \times \{0.951293\} \times \{0.955857\} \times \{0.969745\}] \\
&= 0.006099
\end{aligned}
$$

Hence, average number of surviving children for $T = 4h$ in the free process from equation (1) will be

$$
= (0 \times 0.063887) + (1 \times 0.320258) + (2 \times 0.447957) + (3 \times 0.161818) + (4 \times 0.006099) = 1.72602
$$

2.3 Average Number of Surviving Children for $T = 5h$ in Free Process

Here we have considered $T = 5h$ in the free process. If $h = 1.5$ years, hence the observational period is 7.5 years and there will be 32 probabilities that can be obtained from Yadava and Tiwari (2007) [12]. The average number of surviving children out of total birth in $5h$ will be.

$$
\begin{aligned}
&= [0 \times \Pr(zero\ survivors\ in\ the\ whole\ period)] + [1 \times \Pr(one\ survivor)] \\
&+ [2 \times \Pr(two\ survivors)] + [3 \times \Pr(three\ survivors)] \\
&+ [4 \times \Pr(four\ survivors)] + [5 \times \Pr(five\ survivors)] \qquad (2)
\end{aligned}
$$

Again, we can calculate the probability of zero survivors, one survivor, two survivors, three survivors, four survivors, and five survivors similarly as we have done for $T = 5h$ by classifying the relevant events. By putting all the probabilities in equation (2), the average number of surviving children for $T = 5h$ in the free process will be.

$$
\begin{aligned}
&= (0 \times 0.032676) + (1 \times 0.205805) + (2 \times 0.414150) \\
&+ (3 \times 0.286937) + (4 \times 0.053746) + (5 \times 0.000846) \\
&= 2.11413
\end{aligned}
$$

Table 1: Probability distribution of number of births and number of surviving children in equilibrium birth process and free process for different values of T

Number	Equilibrium Birth Process		Free Process	
	Probability of Number of Births	Probability of Number of Surviving Children	Probability of Number of Births	Probability of Number of Surviving Children
$T = 5h$				
0	0.02840	0.03857	0.02352	0.03268
1	0.19905	0.22899	0.17563	0.20581
2	0.42495	0.42387	0.41019	0.41415
3	0.29687	0.26169	0.32501	0.28694
4	0.04976	0.04130	0.06458	0.05375
5	0.00009	0.00007	0.00107	0.00085
$T = 4h$				
0	0.06023	0.07556	0.04979	0.06389
1	0.32579	0.35066	0.29278	0.32026
2	0.45826	0.43700	0.46629	0.44796
3	0.15082	0.13268	0.18386	0.16182
4	0.00485	0.00405	0.00730	0.00610

Note: $\lambda = 0.5$, $h = 1.5$ years, West Model Life Table at the mortality level 20

Table 2: Comparison between average number of births and average number of surviving children in equilibrium birth process and free process for different values of *T*

Time	Equilibrium Birth Process		Free Process	
	Average Number of Births	Average Number of Surviving Children	Average Number of Births	Average Number of Surviving Children
T = 5h	2.13905	2.02736	2.23471	2.11413
T = 4h	1.71417	1.63891	1.80614	1.72602

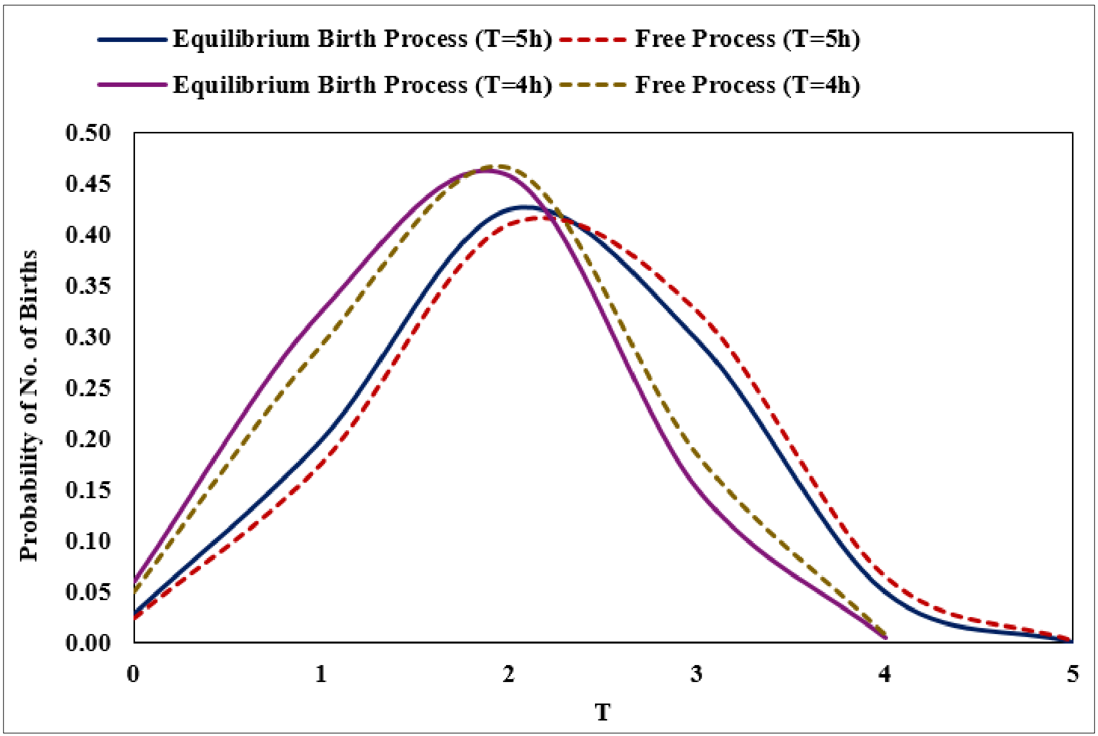

Fig 1: Probability distribution of number of births in equilibrium birth process and free process for different values of *T*

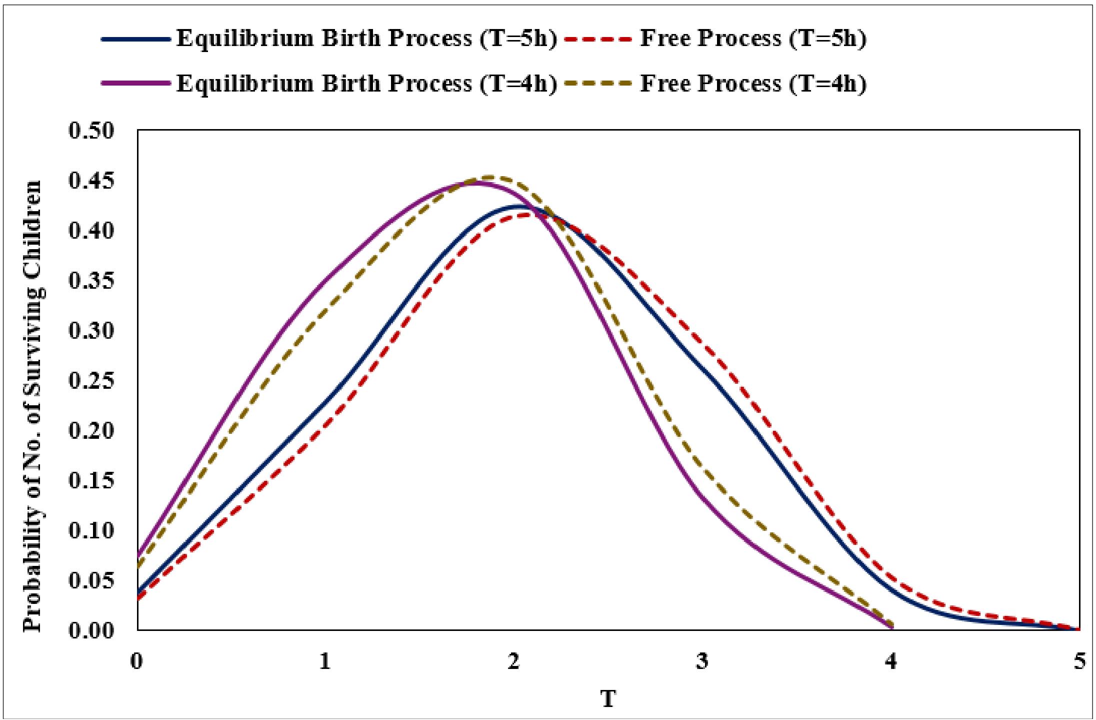

Fig 2: Probability distribution of number of surviving children in equilibrium birth process and free process for different values of *T*

3. Results and Discussion

The probability distribution of the number of births and number of surviving children in the equilibrium birth process as well as in the free process at the mortality level 20 from the West model life table is shown in Table 1. From Table 1, it is observed that probability distribution of births slightly differs between equilibrium birth process and free process, however the length of exposure period is same. When length of period is $T = 5h$, the probability of lower number of births i.e. 0, 1, and 2 is higher in equilibrium birth process, while the probability of higher number of births i.e. 3, 4, and 5 is higher in free process. Similarly when we shortens the length of exposure period i.e. $T = 4h$, the probability of lower number of births i.e. 0, and 1 is higher in equilibrium birth process, while the probability of higher number of births i.e. 2, 3, and 4 is higher in free process (Fig 1). The similar pattern is observed for probability distribution of surviving children also (Fig 2).

The average number of births and the average number of surviving children in the equilibrium birth process and the free process is given in Table 2. It is observed that, within the same length of exposure period more births are likely to occur in free process in comparison to equilibrium birth process. For $T = 5h$, average number of births and average number of surviving children in free process is more than equilibrium birth process. The similar result is also observed for $T = 4h$.

4. Conclusion

By comparing the probability distribution of the number of births in equilibrium birth process and free process, it is observed that probability distribution of births is not same for the equilibrium birth process and the free process irrespective the length of period T. However, the probability for more number of births (i.e. 3, 4, 5 in $T = 5h$ and 2, 3, 4 in $T = 4h$) in the free process is higher than the equilibrium birth process for the period $T = 5h$ as well as for the period $T = 4h$. It justifies the assumption that the females are exposed to the risk of conception in the free process. If we consider the average number of births and the average number of surviving children, a difference has been observed between the equilibrium birth process and the free process. In the free process, average number of births as well as average number of surviving children is higher than the equilibrium birth process. However, the survival ratio remains almost same for the both processes.

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