International Journal of Statistics and Applied Mathematics 2024; 9(3): 34-39

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2024; 9(3): 34-39

https://www.mathsjournal.com
Received: 11-03-2024
Accepted: 14-04-2024

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Classes of univalent functions having fixed point

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DOI: https://doi.org/10.22271/maths.2024.v9.i3a.1728

Abstract
In this paper we investigated the new subclasses of univalent functions with negative coefficient having fixed point. Necessary and sufficient condition for concern class is obtained. Several geometric properties like growth theorem, coefficient estimate, convexness, extreme point theorem has been examined

Keywords: Univalent functions, negative coefficients, fixed point

Introduction

Let N be class of all analytic functions normalized with conditions f (0) =0 and $f'(0) = 1$ in the form f (z) = z - $\sum_{k=2}^{\infty} a_k z^k$, on open unit disc D = {z: $|z| < 1$}.Let N^- is the subclass of N, consist of functions which are in the form.

f (z) = z - $\sum_{k=2}^{\infty} a_k z^k$, $a_k \geq 0$ (1.1)

on open unit disc D = {z: $|z| < 1$}.We have studied the class $K_{\lambda}^{\eta}(\xi, \alpha, \beta, \partial, \lambda, l, u)$. In concern with this class we obtained new class $K_{\lambda}^{\eta}(\xi, \alpha, \beta, \partial, \lambda, l, u, z_0)$ for which f (z_0)=z_0. Silverman [7] obtained the subclasses of starlike and convex functions. Namely $L^*(\sigma)$ and $H^*(\sigma)$. Silverman [8] provided the new classes $L_0^*(\sigma, z_0)$ and $L_1^*(\sigma, z_0)$. These classes consist of the functions in the form f (z) = a_1z - $\sum_{k=2}^{\infty} a_k z^k$.

$L_0^*(\sigma, z_0)$ satisfies $a_k \geq 0$, f (z_0) = z_0 ($z_0 \in (-1, 1)$, $z_0 \neq 0$)

$L_1^*(\sigma, z_0)$ satisfies $a_k \geq 0, f'(z_0) = 1$ $z_0 \in (-1, 1)$

[16] studied the class $L_n^*(\alpha, \beta, \gamma, z_0)$ with certain restriction on α, β, γ. This class is collection of functions in $L_n^*(\alpha, \beta, \gamma)$ which fixes point z_0. Opoola [15] have defined derivative operator. It is describe as given below.

Definition 1.1

The Opoola differential operator for f (z) in N is denoted by $D^n(\mu, \alpha, \zeta)$ f (z) = f(z) (1.2)

$D^1(\mu, \alpha, \zeta)$f(z)= zD_ζf(z)=z $\zeta f'$(z)-z(α- μ) ζ +(1+ α- $\mu - 1)\zeta$)f(z) (1.3)

D^1 (μ, α, ζ) f (z) = z D_ζ(zD_ζf (z)) (1.4)

$D^n(\mu, \alpha, \zeta)$f(z)= zD_ζ($D^{n-1}(\mu, \alpha, \zeta)$), n ∈ {1,2,3,……} (1.5)

After some calculation we have: $(1 + (k + \alpha - \mu - 1)\zeta)^n$

$D^n(\mu, \alpha, \zeta)$ f(z)= z + $\sum_{k=2}^{\infty}(1 + (k + \alpha - \mu - 1)\zeta)^n a_k z^k$ (1.6)

Where $\zeta \geq 0, 0 \leq \mu \leq \alpha$

Also, [6] has defined the following operator, known as Ruschweyh differential operator.

Definition 1.2
For f ε N, [3] has defined following Ruschweyh differential operator.

R^n: N→N defined by

R^n(f (z)) = $\frac{z}{(1-z)^{n+1}}$. f (z) n ∈ $\mathbb{N} \cup \{0\}$

= z + $\sum_{k=2}^{\infty} {}^{n+k-1}_{n}C\ a_k z^k$ (z∈U) (1.7)

Where (.) is hadmard product defined in [1].
We note that R^0 f (z) =f (z), R' f (z) =zf '(z)

2. Class K (O_σ^τ $e, \delta, \vartheta, \zeta, \alpha, \mu$)
Thange [17] introduced the class K (O_σ^τ $e, \delta, \vartheta, \zeta, \alpha, \mu$) associated with Ruopoola derivative operator. We generalized this class to K (F_σ^τ $e, \delta, \vartheta, \zeta, \alpha, \mu$) having more generalized Ruopoola derivative operator. It is obtained by making convex combination of Ruschweyh and Opoola derivative operator.

Definition 2.1
For f ε N, we define the Ruopoola derivative operator F ^{n} as follow

O_t^n(f (z))= (1-t) ($D^n(\mu, \alpha, \zeta)$ f(z)) + t R ^{n}f(z) t∈ [0,1] and n ∈ $\mathbb{N} \cup \{0\}$

= z+ $\sum_{k=2}^{\infty}([1+(k+\alpha-\mu-1)\zeta]^\tau(1-\sigma)+\sigma^{\tau+k-1}_{\tau}C)a_k z^k$ (2.1)

We noted that F_t^0 f(z)=f(z), F_0^nf(z)= R ^{n}f(z) and F_1^nf(z)= D ^{n}f(z)

Definition 2.2
A class K (O_σ^τ $e, \delta, \vartheta, \zeta, \alpha, \mu$) is collection of functions in N^- having.

$$\left| \frac{\frac{z(O_\sigma^\tau(f))'}{(O_\lambda^n(f)}-1}{2e\left(\frac{z(O_\sigma^\tau(f))'}{O_\sigma^\tau(f)}-\delta\right)-\left(\frac{z(O_\sigma^\tau(f))'}{O_\sigma^\tau(f)}-1\right)} \right| < \vartheta \quad (2.2)$$

$Here\ 0 \le \delta < \frac{1}{2e}, 0 < \vartheta \le 1, \frac{1}{2} \le e \le 1, \tau > -1, 0 \le \mu \le \alpha, n\ \varepsilon\ \mathbb{N} \cup \{0\}$.

In our first attempt for this section we obtained the necessary and sufficient condition for the function in the class K (O_σ^τ $e, \delta, \vartheta, \zeta, \alpha, \mu$).

Theorem 2.1
If f (z) = z - $\sum_{k=2}^{\infty} a_k z^k$ is in N, then f ε K (O_σ^τ $e, \delta, \vartheta, \zeta, \alpha, \mu$) if and only if $\sum_{k=2}^{\infty}([1+(k+\alpha-\mu-1)\zeta]^\tau(1-\sigma)+\sigma^{\tau+k-1}_{\tau}C)(2e\vartheta(k-\delta)+(k-1)(1-\vartheta))a_k \le 2\, e\, \vartheta$(1-$\delta$).

Extremal is obtained for the function in the class which are in form

$f_k(z)$ = z- $\frac{2\,e\,\vartheta(1-\delta).}{([1+(k+\alpha-\mu-1)\zeta]^\tau(1-\sigma)+\sigma^{\tau+k-1}_{\tau}C))(2e\vartheta(k-\delta)+(k-1)(1-\vartheta)}z^k$ (1.1)

Proof. Let f (z) = z - $\sum_{k=2}^{\infty} a_k z^k$ is in N^-. *and* e_k= $1+(k+\alpha-\mu-1)\zeta$.
Assume that
$\sum_{k=2}^{\infty}([1+(k+\alpha-\mu-1)\zeta]^\tau(1-\sigma)+\sigma^{\tau+k-1}_{\tau}C)(2e\vartheta(k-\delta)+(k-1)(1-\vartheta))a_k \le 2\, e\, \vartheta$(1-$\delta$)

$$z(O_\sigma^\tau(f))' - (O_\sigma^\tau(f)) = \sum_{k=2}^{\infty}([1+(k+\alpha-\mu-1)\zeta]^\tau(1-\sigma)+\sigma^{\tau+k-1}_{\tau}C)(1-k)\, a_k z^k$$

$$z(O_\sigma^\tau(f))' - \delta(O_\sigma^\tau(f)) = \sum_{k=2}^{\infty}([1+(k+\alpha-\mu-1)\zeta]^\tau(1-\sigma)+\sigma^{\tau+k-1}_{\tau}C)(\delta-k)\, a_k z^k$$

$$\therefore \left| \frac{\frac{z(O_\sigma^\tau(f))'}{(O_\lambda^n(f)}-1}{2e\left(\frac{z(O_\sigma^\tau(f))'}{O_\sigma^\tau(f)}-\delta\right)-\left(\frac{z(O_\sigma^\tau(f))'}{O_\sigma^\tau(f)}-1\right)} \right| = \left| \frac{z(O_\sigma^\tau(f))'-(O_\sigma^\tau(f))}{2e(z(O_\sigma^\tau(f))'-\delta(O_\sigma^\tau(f)))-z(O_\sigma^\tau(f))'-(O_\sigma^\tau(f))} \right|$$

$$= \left| \frac{z(O_\sigma^\tau(f))'-(O_\sigma^\tau(f))}{2e(z(O_\sigma^\tau(f))'-\delta(O_\sigma^\tau(f)))-z(O_\sigma^\tau(f))'-(O_\sigma^\tau(f))} \right|$$

$$= \left| \frac{\sum_{k=2}^{\infty}[e_k^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C](k-1)a_k z^k}{2e(1-\delta)z-\sum_{k=2}^{\infty}[e_k^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C][2e(k-\delta)-(k-1)]a_k z^k} \right|$$

$$\leq \frac{\sum_{k=2}^{\infty}[e_k^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C](k-1)a_k |z^k|}{2e(1-\delta)|z|-\sum_{k=2}^{\infty}[e_k^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C][2e(k-\delta)-(k-1)]a_k |z^k|}$$

$$\leq \frac{\sum_{k=2}^{\infty}[e_k^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C](k-1)a_k}{2e(1-\delta)-\sum_{k=2}^{\infty}[e_k^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C][2e(k-\delta)-(k-1)]a_k}.$$

This holds, since by “Maximum modulus theorem”, maximum occurs only at the boundary points of unit circle.

Hence $\frac{\sum_{k=2}^{\infty}[e_k^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C](k-1)a_k}{2e(1-\delta)-\sum_{k=2}^{\infty}[e_k^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C][2e(k-\delta)-(k-1)]a_k} < \vartheta$

$$\therefore \left| \frac{\frac{z(O_\sigma^\tau(f))'}{(O_\lambda^n(f)}-1}{2e\left(\frac{z(O_\sigma^\tau(f))'}{O_\sigma^\tau(f)}-\delta\right)-\left(\frac{z(O_\sigma^\tau(f))'}{O_\sigma^\tau(f)}-1\right)} \right| < \vartheta.$$

Therefore, f ε K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu)$

Now to prove if part we assume that f (z) ε K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu)$

$$\therefore \vartheta > \left| \frac{\frac{z(O_\sigma^\tau(f))'}{(O_\lambda^n(f)}-1}{2e\left(\frac{z(O_\sigma^\tau(f))'}{O_\sigma^\tau(f)}-\delta\right)-\left(\frac{z(O_\sigma^\tau(f))'}{O_\sigma^\tau(f)}-1\right)} \right|$$

$$= \left| \frac{z(O_\sigma^\tau(f))'-(O_\sigma^\tau(f))}{2e(z(O_\sigma^\tau(f))'-\delta(O_\sigma^\tau(f)))-z(O_\sigma^\tau(f))'-(O_\sigma^\tau(f))} \right|$$

$$= \left| \frac{\sum_{k=2}^{\infty}[e_k^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C](k-1)a_k z^k}{2e(1-\delta)z-\sum_{k=2}^{\infty}[e_k^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C][2e(k-\delta)-(k-1)]a_k z^k} \right|$$

Since Re {z} $\leq z$

$$\therefore \mathrm{Re}\left\{\frac{\sum_{k=2}^{\infty}([1+(k+\alpha-\mu-1)\zeta]^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C)(1-k)a_k z^k}{2e(1-\delta)z-([1+(k+\alpha-\mu-1)\zeta]^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C))(2e(k-\delta)+(k-1))a_k z^k}\right\} < \vartheta$$

Letting z tends to 1 through positive part of real axis in unit disc,

$$\frac{\sum_{k=2}^{\infty}([1+(k+\alpha-\mu-1)\zeta]^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C)(1-k)a_k z^k}{2e(1-\delta)z-([1+(k+\alpha-\mu-1)\zeta]^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C))(2e(k-\delta)+(k-1))a_k z^k} < \vartheta$$

$$\sum_{k=2}^{\infty}([1+(k+\alpha-\mu-1)\zeta]^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C)(2e\vartheta(k-\delta)+(k-1)(1-\vartheta))a_k \leq 2\, e\, \vartheta(1\text{-}\delta).$$

Hence proved.

Example 1.1: If f ε K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu)$ then for k ≥ 2

$$a_k \leq \frac{2e\vartheta(1-\delta)}{([1+(k+\alpha-\mu-1)\zeta]^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C))(2e\vartheta(k-\delta)+(k-1)(1-\vartheta))}$$

Extremal is obtained for the function in the form (1.1)

3. Main Results

In this section we introduced the class K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$. We studied the coefficient bounds, growth theorem, and distortion theorem for this class.

Definition 3.1. The class K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$ consist of functions in the class K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu)$ for which f (z_0)= z_0

Theorem 3.1. A function in the form (1.1) is in K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$ then

$$\sum_{k=2}^{\infty}\left(\frac{B_k}{2e\vartheta(1-\delta)} - z_0^{k-1}\right)a_k \leq 1 \quad (3.1)$$

Here B_k= $([1+(k+\alpha-\mu-1)\zeta]^{\tau}(1-\sigma)+\sigma^{\tau+k-\frac{1}{\tau}}C))(2e\vartheta(k-\delta)+(k-1)(1-\vartheta))$

Converse is true if $\sum_{k=2}^{\infty} z_0^{k-1}\, l_k$=0. (3.2)

Proof. Suppose f ε K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$

∴ f (z_0)= z_0

∴ $z_0 - \sum_{k=2}^{\infty} z_0^{k-1}\, l_k = z_0 \Rightarrow$ 1- $\sum_{k=2}^{\infty} z_0^{k-1}\, l_k = 1$

∴ $\sum_{k=2}^{\infty} z_0^{k-1}\, l_k = 0$

Now f ε K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu) \Rightarrow \sum_{k=2}^{\infty} \frac{B_k}{2e\vartheta(1-\delta)} l_k \le 1$

∴ $\sum_{k=2}^{\infty} \frac{B_k}{2e\vartheta(1-\delta)} l_k - \sum_{k=2}^{\infty} z_0^{k-1}\, l_k \le 1$

∴ $\sum_{k=2}^{\infty} \left(\frac{B_k}{2e\vartheta(1-\delta)} - z_0^{k-1}\right) a_k \le 1$

Now to prove the converse we assume that (3.1) and (3.2) hold

∴ $\sum_{k=2}^{\infty} \left(\frac{B_k}{2e\vartheta(1-\delta)} - z_0^{k-1}\right) a_k \le 1$

∴ $\sum_{k=2}^{\infty} \frac{B_k}{2e\vartheta(1-\delta)} a_k - \sum_{k=2}^{\infty} z_0^{k-1}\, a_k \le 1$

$$\therefore \sum_{k=2}^{\infty} \frac{B_k}{2e\vartheta(1-\delta)} a_k \le 1$$

∴ f ε K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu)$

Suppose $\sum_{k=2}^{\infty} z_0^{k-1}\, a_k = 0$

∴ 1- $\sum_{k=2}^{\infty} z_0^{k-1}\, a_k$=1

∴ z_0- $\sum_{k=2}^{\infty} z_0^{k}\, a_k$=$z_0$

∴ $f(z_0) = z_0$

f ε K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$

Example 3.1: A function in the form (1.1) is in K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$, then.

$a_k \le \frac{2e\vartheta(1-\delta)}{B_k - 2e\vartheta(1-\delta) z_0^{k-1}}$

Here, B_k= $([1 + (k + \alpha - \mu - 1)\zeta]^\tau (1-\sigma) + \sigma^{\tau+k-\frac{1}{\tau}C}))(2e\vartheta(k-\delta) + (k-1)(1-\vartheta))$

Equality is obtained for the function in K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$ in the form.

f_k(z) = z- $\frac{2e\vartheta(1-\delta)}{B_k - 2e\vartheta(1-\delta) z_0^{k-1}}\ z^{k-1}$ (k ≥ 2)

Theorem 3.2: A function in the form (1.1) is in K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$, then for 0 ≤ |z| = ρ < 1

ρ- $\rho^2 \frac{2e\vartheta(1-\delta)}{B_2 - 2e\vartheta(1-\delta) z_0}$ ≤ | f (z) | ≤ ρ+$\rho^2 \frac{2e\vartheta(1-\delta)}{B_2 - 2e\vartheta(1-\delta) z_0}$

Where, B_k= $([1 + (k + \alpha - \mu - 1)\zeta]^\tau (1-\sigma) + \sigma^{\tau+k-\frac{1}{\tau}C}))(2e\vartheta(k-\delta) + (k-1)(1-\vartheta))$

Proof: Given that a function f in the form (1.1) is in K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$,
From equation (3.1)

$$(B_2 - 2e\vartheta(1-\delta) z_0) \sum_{k=2}^{\infty} l_k \le \sum_{k=2}^{\infty} (B_k - 2e\vartheta(1-\delta) {z_0}^{k-1}) l_k$$

$\leq 2e\vartheta(1-\delta)$

$\therefore \sum_{k=2}^{\infty} l_k \leq \frac{2e\vartheta(1-\delta)}{B_2-2e\vartheta(1-\delta)z_0}$

Therefore,

$| f(z) | \leq |z| + |z|^k \sum_{k=2}^{\infty} | l_k | \leq \rho+\rho^2 \sum_{k=2}^{\infty} | l_k |$

$\leq \rho+\rho^2 \frac{2e\vartheta(1-\delta)}{B_2-2e\vartheta(1-\delta)z_0}$

On the same way

$| f(z) | \geq |z| - |z|^k \sum_{k=2}^{\infty} | l_k | \geq \rho+\rho^2 \sum_{k=2}^{\infty} | l_k |$

$\leq \rho\text{-}\rho^2 \frac{2e\vartheta(1-\delta)}{B_2-2e\vartheta(1-\delta)z_0}$

Extremal is obtained for the function in K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$, which are in the form.

$f(z)= z- \frac{2e\vartheta(1-\delta)}{B_k-2e\vartheta(1-\delta)z_0^{t-1}} z^2$, $t \geq 2$

at z= $\rho, \rho e^{i\pi}$

In our next theorem we found the radius of convexness for the class K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$

Theorem 3.3. The radius of convexness for the class K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$ is R,
Where:

R= $inf_k \left(\frac{B_k}{k^2 2e\vartheta(1-\delta)}\right)^{\frac{1}{(k-1)}}$ (k ≥ 2)

Proof. Let f (z) ε K $(O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0)$.
To find the radius of convexness, we assumed f (z) is convex in |z| < R.
We want to find the value of R such that.

$R\left\{\frac{zf''(z)}{f'(z)} + 1\right\} > 0$, |z|<R

But,

$\left|\frac{zf''(z)}{f'(z)} + 1 - 1\right| < 1 \Rightarrow R\left\{\frac{zf''(z)}{f'(z)} + 1\right\} > 0$

Now, f (z) = z - $\sum_{k=2}^{\infty} a_k z^k$

$zf''(z) = -\sum_{k=2}^{\infty} k(k-1)a_k z^{k-1}$

$\left|\frac{zf''(z)}{f'(z)}\right| = \left|\frac{\sum_{k=2}^{\infty} k(k-1)a_k z^{k-1}}{1-\sum_{k=2}^{\infty} k a_k z^{k-1}}\right| \leq \frac{\sum_{k=2}^{\infty} k(k-1)a_k |z^{k-1}|}{1-\sum_{k=2}^{\infty} k a_k |z^{k-1}|}$

$\frac{\sum_{k=2}^{\infty} k(k-1)a_k |z|^{k-1}}{1-\sum_{k=2}^{\infty} k a_k |z|^{k-1}} < 1 \Rightarrow \left|\frac{zf''(z)}{f'(z)}\right| < 1$

Assume that z_0

$\frac{\sum_{k=2}^{\infty} k(k-1)a_k |z|^{k-1}}{1-\sum_{k=2}^{\infty} k a_k |z|^{k-1}} < 1$

$\Rightarrow \sum_{k=2}^{\infty} k(k-1)a_k |z|^{k-1} < 1 + \sum_{k=2}^{\infty} a_k {z_0}^{k-1} \text{-} \sum_{k=2}^{\infty} k a_k |z|^{k-1}$

$\Rightarrow \sum_{k=2}^{\infty} k^2 a_k |z|^{k-1} \text{-} \sum_{k=2}^{\infty} a_k {z_0}^{k-1} <$

$\Rightarrow \sum_{k=2}^{\infty} (k^2 |z|^{k-1} - {z_0}^{k-1})\, a_k < 1$ (3.3)

As f (z) ε K ($O_\sigma^\tau\, e, \delta, \vartheta, \zeta, \alpha, \mu, z_0$). Therefore using (3.3) we have

$$\sum_{k=2}^{\infty}\left(\frac{B_k}{2e\vartheta(1-\delta)} - z_0^{k-1}\right) a_k \le 1$$

Inequality (4.7) is true if

$$\sum_{k=2}^{\infty}(k^2|z|^{k-1} - {z_0}^{k-1})\, a_k < \sum_{k=2}^{\infty}\left(\frac{B_k}{2e\vartheta(1-\delta)} - z_0^{k-1}\right) a_k$$

That is $k^2|z|^{k-1} - {z_0}^{k-1} < \frac{B_k}{2e\vartheta(1-\delta)} - z_0^{k-1} \Rightarrow k^2|z|^{k-1} < \frac{B_k}{2e\vartheta(1-\delta)}$

Hence $|z| < \left(\frac{B_k}{2e\vartheta k^2(1-\delta)}\right)^{\frac{1}{k-1}}$.

Therefore radius of convexness is

$$R = inf_k \left(\frac{B_k}{k^2 2e\vartheta(1-\delta)}\right)^{\frac{1}{(k-1)}}$$

Conclusion

This paper delves into the investigation of subclasses of univalent functions with negative coefficients having a fixed point. Through rigorous mathematical derivations, the necessary and sufficient conditions for the concerned class are derived. The paper explores various geometric properties such as the growth theorem, coefficient estimates, convexity, and extreme point theorem within this context. The findings contribute to the understanding of univalent functions with negative coefficients and their fixed points, shedding light on their intricate properties and providing valuable insights into this area of mathematical analysis.

4. References

1. Juma AR, Kulkarni SR. Some problems connected with geometry of univalent & multivalent functions [PhD thesis]. University of Pune; c2008.
2. Goodman AW. An invitation to the study of univalent and multivalent function. Int. J Math Math Sci. 1979;2:163-186.
3. Al-Oboudi FM. On univalent functions defined by generalized salagean operator. Int. J Math Math Sci. 1999;27:1429-1436.
4. Bshouty D. A note on Hadamard products of univalent functions. Proc Amer Math Soc. 1980;80:271-272.
5. Irmak H, Cetin OF. Some theorems involving inequalities on p valent functions. Turk J Math. 1999;23:453-459.
6. Irmak H, Raina RK. The starlikeness and convexity of multivalent functions. Revista Matematica Complutense. 2003;16(2):391-398.
7. Silvermann HS. Univalent functions with negative coefficient. Proc Amer Math Soc. 1975;37(2):517-520.
8. Silvermann HS. Extreme points of univalent functions with two fixed points. Trans Amer Math Soc. 1976;219:387-395.
9. Jack IS. Functions starlike & convex of order ∝. J London Math Soc. 1971;2(3):469-474.
10. Littlewood JE. On inequalities in the theory of functions. Proc London Math Soc. 1925;23:481-519.
11. Hadamard J. Thorme sur les series entires (French). Acta Math. 1898;22:55-63.
12. Nunokawa M, Sukol J. Condition for starlikness of multivalent functions. Results in Mathematica; c2017.
13. Robertson MS. Applications of Lemma of Fejerto typically real functions. Proc Amer Math Soc. 1950;1:555-561.
14. Duren PL. Univalent functions. Springer-Verlag, New York, USA; c1983.
15. Timothy OO. On a subclass of univalent functions defined by a generalized differential operator. Int J Math Anal. 2017;11(18):869-876.
16. Joshi SB, Joshi SS, Pawar H. On subclass of starlike functions with fixed points. J Ind Acad Math. 2015;37(1):101-111.
17. Thange TG, Jadhav SS. On subclasses of univalent functions having negative coefficient using Rusal differential operator. Advanc. in Math Sci. J, 2020, 9(4).