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Finite single integral representation for the multivariable generalized polynomial Set $A_n\{(x_m), y\}$

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Abstract
In the present paper, an attempt has been made to express a Finite Single Integral Representation for the Multivariable Generalized polynomial set $An\{(xm), y\}$. Many interesting new results may be obtained as particular cases on separating the parameters.
AMS Subject Classification: Special function-33 (C)

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Introduction
We obtained the multivariable generalized polynomial set $An\{(xm), y\}$ by means of generating relation

$$e^{\mu t^e} F\begin{bmatrix} p_z, (a_g); \\ \lambda y^{-\mu, e_1} t^{e_1} \\ (b_\square); \end{bmatrix}$$

$$\times F\left[\begin{matrix}(A_r); (C_p); (\alpha_{u_m}) \\ (B_s); (D_q); (\beta_{v_m})\end{matrix} \lambda x^d t, \lambda_2 x_2^{\square_2} y^{-\mu_2 \square_2} \dots \dots \lambda_m x_m^{\square_m} t^{\square_m}\right]$$

$$= \sum_{n=0}^{\infty} A_{n,e;e_1;e_2;\dots\dots e_m;(b_\square);(B_s);(D_q);(\beta_{v_m})}^{\nu;\mu;\alpha;\mu_1;\mu_2;\lambda;\lambda_1;\lambda_2\dots\dots\lambda_m,d;(a_g);(A_r);(C_p);(\alpha_{u_m})} \{(x_m), y\} t^n \qquad \dots (1.2)$$

where ν, μ, μ_1, μ_2, λ, λ_1, λ_2, ……λ_m, d are real and e, e_1 are non-negative integer and e_2 …. e_m are natural number.

The left hand side of (1.2) are contains the product of generalized hypergeometric function and Lauricella function in the notation of Burchnall and Chaundy [2]. The polynomial set contains number of parameters for simplicity we shall denote

$$A_{n,e;e_1;e_2;\dots\dots e_m;(b_\square);(B_s);(D_q);(\beta_{v_m})}^{\nu;\mu;\alpha;\mu_1;\mu_2;\lambda;\lambda_1;\lambda_2\dots\dots\lambda_m,d;(a_g);(A_r);(C_p);(\alpha_{u_m})} \{(x_m), y\} \text{ by } A_n\{(x_m), y\}.$$

where n denotes the order of the polynomial set.

2. Notations
A. (i) $(n) = 1, 2, \dots\dots n-1, n$.
(ii) $(a_p) = a_1, a_2, a_3, \dots\dots a_p$.
(iii) $(a_p; i) = a_1, a_2, a_3, \dots\dots a_{i-1}, a_{i+1} \dots\dots a_p$.

B. (i) $[(a_p)] = a_1, a_2, a_3 \dots\dots a_p$.
(ii) $[(a_p)]_n = \prod_{i=1}^{p} (a_i)_n = (a_1)_n (a_2)_n (a_3)_n \dots (a_p)_n$

C. (i) $\Delta(a;b) = \frac{b}{a}, \frac{b+1}{a} + \cdots \ldots \frac{b+a-1}{a}$.

(ii) $\Delta(a(1);b) = \frac{b}{a}, \frac{b+1}{a}, \frac{b+2}{a}, \ldots \ldots \frac{b+a-2}{a}$

(iii) $\Delta(a;\, b \pm c \pm d) = \Delta(a;\, b + c + d), \Delta(a;\, b + c - d)$,
$\Delta(a;\, b - c + d), \Delta(a;\, b - c - d)$,

D. (i) $\Delta_k[a;b] = \prod_{r=1}^{a}\left(\frac{b+r-1}{a}\right)_k = \left(\frac{b}{a}\right)_k \left(\frac{b+1}{a}\right)_k \cdots \left(\frac{b+a-1}{a}\right)_k$.

(ii) $\Delta_k[a(1);b] = \left(\frac{b}{a}\right)_k \left(\frac{b+1}{a}\right)_k \cdots \left(\frac{b+a-2}{a}\right)_k$.

(iii) $\Delta_k[m;(a_p)] = \prod_{i=1}^{b}\prod_{r=1}^{m}\left(\frac{a_i+r-1}{m}\right)_k$.

E. (i) $\Gamma[(a_p)] = \prod_{i=1}^{p}(a_i)$.

(ii) $\Gamma[(a_p);(s)] = \prod_{i=s+1}^{p}(a_i)$.

(iii) $\Gamma[(a;b)] = \prod_{r=1}^{a}\Gamma\left(\frac{b+r-1}{a}\right)$

(iv) $\Gamma[\Delta(m);(a_p)] = \prod_{i=1}^{p}\prod_{r=1}^{m}\Gamma\left(\frac{(a_i)+r-1}{m}\right)$

F. (i) $\Gamma(a \pm b) = \Gamma(a+b)\Gamma(a-b)$.

(ii) $\Gamma_{**}\,(a+b) = \Gamma(a+b)\Gamma(a+b)$

$$M = \frac{[(A_r)]_n[(C_p)]_n(\lambda x_1^d)^n}{[(B_s)]_n[(D_q)]_n n!}$$

3. Theore

For $e_2 > 1, e_3 > 1 \ldots\ldots e_m > 1$

$$A_n\{(x_m), y\} = \frac{\Gamma(1+a)\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}{\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-b-c-d)}$$

$$\times \int_0^{'} x_1^{d-1}(1-x_1)^{a-2d}\; {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ x_1 \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right]$$

$$\times F_{r+p\,:\,h\,:\,\beta_{u_1},\beta_{u_2}\ldots\ldots\beta_{u_m}}^{1+s+q\,:\,g\,:\,\alpha_{u_1},\alpha_{u_2}\ldots\ldots\alpha_{u_m}}\left[\begin{matrix}(-n):e,e_1,e_2;\ldots\ldots e_m] \\ \overline{}\end{matrix}\right.,$$

$$\begin{matrix}[(1-(B_s)-n):e,e_1,e_2-1\ldots e_m-1],[(1-(D_q)-n):e,e_1,e_2;\ldots\ldots e_m][(a_g);1] \\ [(1-(A_r)-n):e,h_1,e_2-1\ldots e_m-1],[(1-(C_p)-n):e,e_1,e_2;\ldots\ldots e_m][(b_h);1]\end{matrix}$$

$$\left[\begin{matrix}[(\alpha_{u_1}):1],[(\alpha_{u_2}):1]\ldots\ldots[(\alpha_{u_m}):1][(v_z):1],[(1+a-2d):2][1+a-b-c-d;1], \\ [(\beta_{v_1}):1],[(\beta_{v_2}):1]\ldots\ldots[(\beta_{v_m}):1],[(d):1][(1+a-c-d):1][(1+a-c-d):1],\end{matrix}\right.$$

$$\frac{\mu(-1)^{e(r+s+p+q+g+h+1)}}{(\lambda x_1^d)^e}, \frac{\lambda_1(-1)^{e_1(r+s+p+q+g+h+1)}}{(\lambda x_1^d y^{\mu_1})^{e_1}}, \ldots\ldots$$

$$\left.\frac{\lambda_2 x_2^{e_2}(-1)^{e_2(r+s+p+q+g+h+1)+r+s}}{(\lambda x_1^d y^{\mu_2})^{e_2}}, \frac{\lambda_m x_m^{em}(-1)^{em(r+s+p+q+g+h+1)+r+s}}{(\lambda x_1^d)^{em}}\right] dx \tag{3.1}$$

Provided that $Re(d)$, $Re\,(a-2d) > -1$ and $Re\,(b+c+d-a) > -1$.

Proof: we have

$$I_1 = \int_0^{'} x_1^{d-1}(1-x_1)^{a-2d}\; {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ x_1 \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right]$$

$$\times \sum_{k=0}^{\left[\frac{n}{e}\right]} \sum_{k_1=0}^{\left[\frac{n-ek}{e_1}\right]} \sum_{k_2=0}^{\left[\frac{n-ek-e_1k_1}{e_2}\right]} \ldots \ldots \sum_{k_m=0}^{\left[\frac{n-ek-e_1k_1\ldots\ldots-e_{m-1}k_{m-1}}{e_m}\right]}$$

$$\times \frac{[(A_r)]_{n-ek-e_1k_1-(e_2-1)k_2-\cdots\ldots-(e_m-1)k_m}[(C_p)]_{n-ek-e_1k_1-e_2k_2-\cdots\ldots-e_mk_m}}{[(B_s)]_{n-ek-e_1k_1-(e_2-1)k_2-\cdots\ldots-(e_m-1)k_m}[(D_q)]_{n-ek-e_1k_1-e_2k_2-\cdots\ldots-e_mk_m}}$$

$$\times \frac{[(a_g)]_{k_1}(\nu_z)_{k_1}\mu^k \lambda_1^{k_1}(\lambda_2 x_2^{e_2})^{k_2}\ldots(\lambda_m x_m^{e_m})^{k_m}(\lambda x_1^d)^{n+ek+e_1k_1+e_2k_2+\cdots\ldots+e_mk_m}}{[(b_h)]_{k_1}\, k!\, k_1!\, k_2!\, y^{\mu_1e_1k_1+\mu_2e_2k_2}\, k_m!\,(n-ek-e_1k_1-e_2k_2\ldots\ldots e_mk_m)!}$$

$$\times \frac{(1-a-2d)_{2k_1}(1+a-b-c-d)_{k_1}}{(d)_{k_1}(1+a-b-d)_{k_1}(1+a-c-d)_{k_1}}$$

$$\times \frac{[(\alpha_{u_1})]_{k_1}[(\alpha_{u_2})]_{k_2}\ldots\ldots[(\alpha_{u_m})]_{k_m}dx_x}{[(\beta_{v_1})]_{k_1}[(\beta_{v_2})]_{k_2}\ldots\ldots[(\beta_{v_m})]_{k_m}}$$

$$= \int_0^{'} x_1^{d+k_1-1}(1-x_1)^{a-2d-2k_1}\; {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ x_1 \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right]$$

$$\times \sum_{k=0}^{\left[\frac{n}{e}\right]} \sum_{k_1=0}^{\left[\frac{n-ek}{e_1}\right]} \sum_{k_2=0}^{\left[\frac{n-ek-e_1k_1}{e_2}\right]} \ldots \ldots \sum_{k_m=0}^{\left[\frac{n-ek-e_1k_1\ldots\ldots-e_{m-1}k_{m-1}}{e_m}\right]}$$

$$\times \frac{[(A_r)]_{n-ek-e_1k_1-(e_2-1)k_2-\cdots\ldots-(e_m-1)k_m}[(C_p)]_{n-ek-e_1k_1-e_2k_2-\cdots\ldots-e_mk_m}[(a_g)]_{k_1}(\nu_z)_{k_1}\mu^k\lambda_1^{k_1}}{[(B_s)]_{n-ek-e_1k_1-(e_2-1)k_2-\cdots\ldots-(e_m-1)k_m}[(D_q)]_{n-ek-e_1k_1-e_2k_2-\cdots\ldots-e_mk_m}[(b_h)]_{k_1}\, k!\, k_1!\, k_2!}$$

$$\times \frac{(\lambda_2 x_2^{e_2})^{k_2}\ldots(\lambda_m x_m^{e_m})^{k_m}(\lambda x_1^d)^{n+ek+e_1k_1+e_2k_2+\cdots\ldots+e_mk_m}}{y^{\mu_1e_1k_1+\mu_2e_2k_2}\, k_m!\,(n-ek-e_1k_1-e_2k_2\ldots\ldots e_mk_m)!}$$

$$\times \frac{[(\alpha_{u_1})]_{k_1}[(\alpha_{u_2})]_{k_2}\ldots\ldots[(\alpha_{u_m})]_{k_m}(1-a-2d)_{2k_1}(1+a-b-c-d)_{k_1}dx_x}{[(\beta_{v_1})]_{k_1}[(\beta_{v_2})]_{k_2}\ldots\ldots[(\beta_{v_m})]_{k_m}(d)_{k_1}(1+a-b-d)_{k_1}(1+a-c-d)_{k_1}}$$

$$\times \sum_{k=0}^{\left[\frac{n}{e}\right]} \sum_{k_1=0}^{\left[\frac{n-ek}{e_1}\right]} \sum_{k_2=0}^{\left[\frac{n-ek-e_1k_1}{e_2}\right]} \ldots \ldots \sum_{k_m=0}^{\left[\frac{n-ek-e_1k_1\ldots\ldots-e_{m-1}k_{m-1}}{e_m}\right]} \frac{[(A_r)]_{n-ek-e_1k_1-(e_2-1)k_2-\cdots\ldots-(e_m-1)k_m}}{[(B_s)]_{n-ek-e_1k_1-(e_2-1)k_2-\cdots\ldots-(e_m-1)k_m}}$$

$$\times \frac{[(C_p)]_{n-ek-e_1k_1-e_2k_2-\cdots\ldots-(e_m-1)k_m}[(a_g)]_{k_1}(\nu_z)_{k_1}[(\alpha_{u_1})]_{k_1}}{[(D_q)]_{n-ek-e_1k_1-e_2k_2-\cdots\ldots-(e_m-1)k_m}[(b_h)]_{k_1}\,[(\beta_{v_1})]_{k_1}}$$

$$\times \frac{[(\alpha_{u_2})]_{k_2}\ldots\ldots[(\alpha_{u_m})]_{k_m}\mu^k\lambda_1^{k_1}(\lambda_2 x_2^{e_2})^{k_2}\ldots(\lambda_m x_m^{e_m})^{k_m}(\lambda x_1^d)^{n+ek+e_1k_1+e_2k_2+\cdots\ldots+e_mk_m}}{[(\beta_{v_2})]_{k_2}\ldots\ldots[(\beta_{v_m})]_{k_m}\, k!\, k_1!\, k_2!\, y^{\mu_1e_1k_1+\mu_2e_2k_2}k_m!\,(n-ek-e_1k_1-e_2k_2\ldots\ldots e_mk_m)!}$$

$$\times \frac{(1-a-2d)_{2k_1}(1+a-b-c-d)_{k_1}\Gamma(d+k_1)\Gamma(1+a-b)\Gamma(1+a-c)}{(d)_{k_1}(1+a-b-d)_{k_1}(1+a-c-d)_{k_1}\Gamma(1+a)\Gamma(1+a-b-c)}$$

$$\times \frac{\Gamma(1+a-2d-2k_1)\Gamma(1+a-b-c-d-k_1)\Gamma(1+a-b-c-d+k_1)}{\Gamma(1+a-b-d-k_1)\Gamma(1+a-c-d-k_1)}$$

$$= \frac{\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-b-c-d)}{\Gamma(1+a)\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}$$

$$\times M \sum_{k,k_1,k_2\ldots k_m}^{\infty} \frac{[1-(B_s)-n]_{ek+e_1k_1+(e_2-1)k_2+\cdots\ldots+(e_m-1)k_m}}{[1-(A_r)-n]_{ek+e_1k_1+(e_2-1)k_2+\cdots\ldots+(e_m-1)k_m}}$$

$$\times \frac{[1-(D_q)-n]_{ek+e_1k_1+e_2k_2+\cdots\ldots+e_mk_m}[(a_g)]_{k_1}(\nu_z)_{k_1}}{[1-(C_p)-n]_{ek+e_1k_1+e_2k_2+\cdots\ldots+e_mk_m}[(b_h)]_{k_1}}$$

$$\times \frac{[(\alpha_{u_1})]_{k_1}[(\alpha_{u_2})]_{k_2}\ldots\ldots[(\alpha_{u_m})]_{k_m}\mu^k(-1)^{e(r+s+p+q+1+g+h)k}}{[(\beta_{v_1})]_{k_1}[(\beta_{v_2})]_{k_2}\ldots\ldots[(\beta_{v_m})]_{k_m}(\lambda x_1^d)^{ek}k!}$$

$$\times \frac{\lambda_1(\nu_z)_{k_1}(-1)^{e_1(r+s+p+q+g+h+1)k_1}(\lambda_2 x_2^{e_2})^{k_2}(-1)^{e_2\{(r+s+p+q+g+h+1)r+s\}k_2}}{(\lambda x_1^d)^{e_1k_1}y^{\mu_1e_1k_1}(\lambda x_1^d y^{\mu_2})^{e_2k_2}}$$

$$\times \frac{\ldots(\lambda_m x_m^{e_m})^{k_m}(-1)^{\{e_m(r+s+p+q+g+h+1)r+s\}k_m}(-n)_{ek+e_1k_1+e_2k_2+\cdots\ldots+e_mk_m}}{(\lambda x_1^d)^{e_mk_m}} \quad (3.2)$$

The single terminating factor makes all summation in (3.2) runs upto ∞.
Then we finally achieve.

$$= \frac{\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-2d)\Gamma(1+a-b-c-d)\Gamma(1+a-c)A_n\{(x_m),y\}}{\Gamma(1+a)\Gamma(1-a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}$$

Hence the proof.
We have from [3]

$$\int_0^{'} x_1^{d-1}(1-x_1)^{a-2d}\, {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ x_1 \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right]$$

$$= \frac{\Gamma(d)\Gamma(1+a-2d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-b-c-d)}{\Gamma(1+a)\Gamma(1-a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}$$

Provided that $Re(d)$, $Re(a-2d) > -1$ and $Re(b+c+d-a) > -1$.

Particular Cases of (3.1)

(i) If we take $r = 0 = s = p = q = \alpha_{u_1} = \beta_{v_1}$; $g = p = q$; $\mu = 1 = z = e = e_1 = \lambda_1 = \lambda = d = m_1 = \mu_1 = y$; $x_1 = \frac{1}{x}$, we get

$$1F_1(-n; b; x) = \frac{\Gamma(1+a)\,\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}{\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-b-c-d)}$$

$$\times \int_0^{'} x_1^{d-1}(1-x_1)^{a-2d}\, {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ x_1 \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right]$$

$$\times F\left[\begin{matrix} -n, \Delta(2{:}\,1+a-2d), (1+a-c-d)(a_p); \\ x_1 \\ d, 1+a-b-d, 1+a-c-d, (b_q); \end{matrix}\right] dx_1$$

where $1F_1[-n, b; x]$ are the Abdul-Halim and Al-Salam Polynomials [03].

(ii) On putting $r = 0 = s = p = q = u_{\alpha_1} = v_{\beta_1}$; $g = h$; $z = 1 = \mu_1 = d = y = \lambda = \mu = e = e_1 = a_1 = b_1$; $x_1 = y$; $\mu_1 = 1 + \lambda_2$, we get

$$A_n^{\lambda_2}(y) = \frac{\Gamma(1+a)\,\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)y^n}{\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-c-d)n!}$$

$$\times \int_0^{'} x_1^{d-1}(1-x_1)^{a-2d}\, {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ x_1 \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right]$$

$$\times F\left[\begin{matrix} -n, \Delta(2{:}\,1+a-2d), (1+a-c-d)1+\lambda_2; \\ \frac{1}{y} \\ d, 1+a-b-d, 1+a-c-d; \end{matrix}\right] dx_1$$

where $A_n^{\lambda_2}(y)$ are the Srivastava Polynomials [4].

(iii) For $r = 0 = s = p = g = h = \alpha_{u_1} = \beta_{v_1}$; $p = 1 = e = e_1 = d = x_2 = x_m = y = \mu = z = \nu$; $x_1 = \frac{1}{y}$; $c_1 = 1 + \lambda$, $\lambda = -1$, we get

$$A_n^{\lambda}(y) = \frac{\Gamma(1+a)\,\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)(1+\lambda)_n(-1)^n y^n}{n!\Gamma(d)\Gamma(1-a+b)\Gamma(1+a-b-c)\Gamma(1+a-2d)\Gamma(1+a-b-c-d)}$$

$$\times \int_0^{'} x_1^{d-1}(1-x_1)^{a-2d}\, {}_4F_3\left[\begin{matrix} a, 1+\frac{a}{2}, b, c; \\ x_1 \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{matrix}\right]$$

$$\times F\begin{bmatrix} -n, \Delta(2; 1+a-2d), 1+a-b-c-d; \\ -y \\ -\lambda-n, d, 1+a-b-d, 1+a-c-d; \end{bmatrix} dx_1$$

Where $L_n^{(\alpha)}(y)$ are the Srivastava Polynomials [5].

(iv) On making the substitution $r = 0 = p = q = s = g = \alpha_{u_1} = \beta_{v_1}$; $h = 1 = e = e_1 = d = \nu = x = \lambda = \mu_1$; $b_1 = 1 + \alpha$, and $x_1 = \frac{1}{y}$, we get

$$L_n^{(\alpha)}(y) = \frac{\Gamma(1+\alpha)_n \Gamma(1+a)\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)}{n! y^n \Gamma(d)\Gamma(1-a-b)\Gamma(1+a-b-c)\Gamma(1+a-2d)\Gamma(1+a-b-c-d)}$$

$$\times \int_0^{'} x_1^{d-1} (1-x_1)^{a-2d} {}_4F_3\begin{bmatrix} a, 1+\frac{a}{2}, b, c; \\ x_1 \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{bmatrix}$$

$$\times F\begin{bmatrix} -n, \Delta(2; 1+a-2d), 1+a-b-c-d; \\ y \\ 1+\alpha, d, 1+a-b-d, 1+a-c-d; \end{bmatrix} dx_1$$

where $L_n^{(\alpha)}(y)$ are the Laguerre Polynomials.

(v) For $r = 0 = s = p = s = \alpha_{u_1} = \beta_{v_1}$; $q = h = e = e_1 = d = \nu_z = \mu_1$; $\lambda = \frac{1}{2} = \lambda_1, D_1 = \frac{1}{2} = b_1$; and $x_1 = \frac{x-1}{x+1}$, we get

$$T_n(x_1) = \frac{\Gamma(1+a)\,\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-b-c-d)\left(\frac{x-1}{2}\right)^n}{\Gamma(d)\Gamma(1-a-b)\Gamma(1+a-b-c)\Gamma(1+a-2d)\Gamma(1+a-b-c-d)}$$

$$\times \int_0^{'} x_1^{d-1} (1-x_1)^{a-2d} {}_4F_3\begin{bmatrix} a, 1+\frac{a}{2}, b, c; \\ x_1 \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{bmatrix}$$

$$\times F\begin{bmatrix} -n, -\frac{1}{2}-n, \Delta(2; 1+a-2d), 1+a-b-c-d; \\ x_1 \\ \frac{1}{2}, d, 1+a-b-d, 1+a-c-d; \end{bmatrix} dx_1$$

Where, $T_n(x)$ are the Tchebicheffe Polynomials.

(vi) For the value of $r = 0 = s = p = g = \alpha_{u_1} = \beta_{v_1}$; $q = 1 = h = e = e_1 = d = \nu_z = \mu_1$; $\lambda = \frac{1}{2} = \lambda_1, D_1 = \frac{3}{2} = b_1$; and $\frac{x-1}{x+1}$ for x_1, we get

$$U_n(x) = \frac{\Gamma(1+a)\,\Gamma(1+a-b-c)\Gamma(1+a-b-d)\Gamma(1+a-c-d)\left(\frac{x-1}{2}\right)^n}{\Gamma(d)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+a-2d)\Gamma(1+a-b-c(-d)}$$

$$\times \frac{(x-1)!}{n!} \int_0^{'} x_1^{d-1} (1-x_1)^{a-2d} {}_4F_3\begin{bmatrix} a, 1+\frac{a}{2}, b, c; \\ \frac{x+1}{x-1} \\ \frac{a}{2}, 1+a-b, 1+a-c; \end{bmatrix}$$

$$\times F\begin{bmatrix} -n, -\frac{1}{2}-n, \Delta(2; 1+a-2d), 1+a-b-c-d; \\ \frac{x+1}{x-1} \\ \frac{3}{2}, d, 1+a-b-d, 1+a-c-d; \end{bmatrix} dx_1$$

where $U_n(x)$ are the Tchebicheffe Polynomials of Second Kind.

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