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Mean: Reverting logistic Brownian motion with jump diffusion process on energy commodity prices

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Abstract
Models that can describe strike prices of energy commodities that might seem costly to store are best modeled by mean - reversion and jump diffusion processes. Physical characteristics of energy commodities makes it very difficult to store due to their salient features hence there is need to incorporate jumps and mean - reversion to some stochastic volatility models and particularly in this paper, we forecast on logistic Brownian motion. I construct a real option model to predict prices of energy commodities. This study also examines some implications on assumptions that can be portrayed by mean - reverting logistic Brownian motion with jump diffusion process. This study uses Heave - side cover up method, logistic Brownian motion, jump diffusion models and mean - reverting models to derive a pricing process that can be used to predict prices of energy commodities.

Keywords: Jump diffusion, mean-reversion, geometric Brownian motion, logistic Brownian motion, heave-side cover-up

1. Introduction
In financial literature we find that commodities that are much storable are the ones that have their prices modeled. There has been assumptions that all commodities traded are storable which is no longer true as energy commodities like electricity in the recent years has been one good traded asset in most security exchange companies.

The growth of energy commodity markets has been rapid due to new development in re – structuring the supply of electricity and invention of new substitutes to electricity in Kenya. Kenya power markets has increased along-side increase in electricity substitutes with increasing demand of electricity as a result of increasing population. The average volume of units purchased from hydro-power sources and geo-thermal reduced by 13.2 percent from 3,787 GWh to 3, 341 GWh and 3.4 percent from 4, 607 GWh to 4,451 GWh respectively for every year from 2016-2020.

The cost of fuel also increased by an average of Ksh. 9, 434 million from 12, 690 to 22, 124 million each year from 2016-2020. This is as a result of increased usage of thermal sources during each and every year. The volume units generated from geo-thermal plants increased at an average from 1,297 GWh to 2, 165 GWh each and every year from 2016-2020. This result to a percentage increase of 66.9 percent.

On the cost of transmission and distribution there has been an increase at an average from 28, 651 million to 37, 417 million each and every year from 2016-2020. The high cost of operation and maintenance led to the increase and also due to the expanded electricity network facilities.

From the global electricity market trends, reforms inevitably exposes the value of portfolios when generating assets and contract supply that are held in traditional electricity power companies to price market of risks. Asset valuation and risk management generally needs a required in-depth to understand and sophisticate the way we model spot prices of commodities. Electricity generally pose the major challenge for financial researchers and practitioners when modeling its spot price behavior among all energy commodities. This is majorly because of it's distinguish feature that it cannot be stored or inventoried economically once it is generated. More so the supply and demand of bulk energy commodity due to increased population has to be balanced continuously so as to prevent it from collapsing. Electricity spot prices are volatile since their supply and demand cannot be smoothed by inventories.

Few studies exist in financial literature in modeling of electricity in Kenya. This paper therefore investigates the pricing trends when the standard logistic Brownian motion is extended by incorporating mean reversion and jump diffusion processes to come up with spot pricing model. The mean – reverting logistic Brownian motion model with jump diffusion process that is used in this study has a non – instantaneous volatility borrowed from Heston's volatility equation. Andanje *et al.* [1, 2]. This is given by;

$$\frac{dX_t}{X_t(X^*-X_t)} = \alpha\left(\overline{X} - \ln X\right)dt + \sqrt{\upsilon_t}dZ_t + dq \quad (1.1)$$

where;

- $\overline{X}$ is the value around which X_t tends to oscillate, generally known as the level of mean reversion or strike price in the long – run.
- X_t is the price of energy commodity at time t.
- α is the mean reverting speed.
- Z_t is the standard Wiener process such that $dZ_t = \varepsilon_t\sqrt{dt}$ and $\varepsilon_t \sim N(0,1)$.
- υ_t is the non – constant volatility borrowed from Heston' s model of closed for vanilla option.
- q is the jump process generated by Poisson process.

It is from equation (1.1) that we subject the Heave – side cover-up method to derive the spot price option model. Oduor [12, 13, 14].

2. Preliminaries

2.1 Stochastic process

These are uncertain ways that brings about changes in values of variables where we know the distribution process of possible values of variables at any point in time. Stochastic processes generally follows some laws of probability. A stochastic process can be expressed mathematically as $\{X_t : t \in (0,\infty)\}$ which shows collections of random variables in each t over the index set $(0,\infty)$. Stochastic process can be distinguished into two parts, a discrete time stochastic process and a continuous time stochastic process. The discrete time stochastic process has change of variables at a certain fixed points in time while the continuous time stochastic process has certain change of value of variable over a given range.

2.2 Markov Stochastic Process

It occurs where the past history of a value of an asset is not influencing the behavior of the current value of the asset. This is because the present price of the asset already involves all the relevant details from the past history that would have otherwise affected the new price of the underlying variable.

2.3 Wiener Process

Wiener process is a type of Brownian motion where we have a mean rate of change being zero and a variance rate of change being one. It follows the properties given by;

Property 1: Given a small change of time period, ΔW we define

$$\Delta W = \varepsilon\sqrt{\Delta t} \quad (2.1)$$

where $\varepsilon \sim N(0,1)$

Property 2: Given two distinct short time periods, ΔW that are independent, we define;

$$cov\left(\Delta W_i, \Delta W_j\right) = 0 \quad (2.2)$$

where $i \neq j$.

Hull [7]

2.4 Jump Process

It is the abrupt deviation that occurs in value of a derivative from its normal path as a result of non – systematic risk. This may happen as a result of new information that causes positive or negative results in price of the asset. This information may be sector firm oriented. It may also not be of systematic risk but rather of jumps that are non – correlated with the market.

2.5 Options

It is the contract between two investors stating an obligation to buy or sell underlying derivative at a certain price after or before the expiry date. There are two types of options; *call option* and *put option.*

2.5.1 Call option

It is a contract that allows one have a right to buy an underlying derivative at a certain price before or at expiry date.

2.5.2 Put option

It is a contract that allows one have a right to sell an underlying derivative at a certain price before or at expiry date.

2.6 Styles of option contracts

It is the aspect of executing or exercising an option. During the contract, the seller is paid by the buyer a percentage of value at maturity. We have two types of option contracts; European options and American options.

2.6.1 European options

It is an option where the holder of the derivative has a right but not an obligation to buy or to sell an option only on the expiry date at a certain price.

2.6.2 American options

It is an option that gives the holder a right but not obligation to buy or to sell an option on or prior to the maturity date that is to say, any time before maturity date at a certain price.

2.7 Itô Process

It is a generalized Wiener diffusion process in having constant parameters a and b taken as functions of the price of an underlying derivative defined by the variables X and t. It is usually written mathematically by a process of diffusion given by;

$$dX = a(X,t)dt + b(X,t)dW_t \quad (2.3)$$

Where a is the expected drift rate and b is the rate of variance of the Itô process that is always liable to change over a small time interval between t and $t + \Delta t$, this results to change from

X to $X+\Delta X$. Equation (2.3) therefore can be expressed as follows from the resulting small change;

$$\Delta X = a(X,t)\Delta t + b(X,t)\varepsilon\sqrt{\Delta t} \qquad (2.4)$$

The resulting relationship is from a small approximation with $a(X,t)\Delta t$ being the drift and $b^2(X,t)\Delta t$ being the rate of variance.

2.8 Itô Lemma

Stochastic differential equations are best solved by Itô Lemma, where wiener - like differential process are put into mathematical formulation of partial differential equations to obtain solutions of stochastic differential equations. In deriving Itô lemma we consider value of a variable P that follows an Itô process from equation (3), where P is said to have a linear drift rate of a and a percentage variance of b^2 such that from Itô lemma, it is stated that a

A function $G(X,t)$ that can be twice differentiable in X and once in t is an Itô Lemma derived from an Itô process of the form;

$$dG = \left(\frac{\partial G}{\partial X}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial X^2}b^2\right)dt + \frac{\partial G}{\partial X}bdW_t, \qquad (2.5)$$

Where dW_t is the standard Wiener diffusion process, the

Percentage rate of drift G is given by $\left(\frac{\partial G}{\partial X}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial X^2}b^2\right)$

And a percentage rate of variance is given by $\left(\frac{\partial G}{\partial X}\right)^2 b^2 dt$

2.9 European – Logistic – Type option pricing model.

A geometric Brownian motion is said to possess features of a stochastic process if it is in the following form of stochastic differential equation used in the application of stock valuation. It is given by;

$$dX_t = \mu X_t dt + \sigma X_t dW_t, \qquad (2.6)$$

where X_t is the value of the underlying variable or option at time t, μ is a constant which is the expected drift rate or the return percentage of the underlying derivative or option and σ is a constant which is the percentage rate of volatility of value of the underlying asset.

The excess demand function is normally given by;

$$ED(X_t) = QD(X_t) - QS(X_t) \qquad (2.7)$$

where $ED(X_t)$ the excess demand is while $QD(X_t)$ and $QS(X_t)$ denotes the quantity demanded and supplied respectively. This leads to the asset price that follows a logistic Brownian motion derived as;

$$dX_t = \mu X_t(X^* - X_t)dt + \sigma X_t(X^* - X_t)dW_t \qquad (2.8)$$

D. B. Oduor *et al.* [4].

Equation (2.8) can be re – written as;

$$\frac{1}{X_t}\frac{dX_t}{(X^* - X_t)} = \mu dt + \sigma dW_t \qquad (2.9)$$

where X^* is the state of equilibrium market value obtained from the Walrasian price mechanism, X_t is the value of the derivative option at a given time t, μ is a constant which is the expected drift rate or the return percentage of the underlying derivative or option and σ is a constant which is the percentage rate of volatility. In this paper we use $\sqrt{\upsilon_t}$ to denote the non – constant instantaneous volatility borrowed from the Heston's volatility model.

3. Main results

3.1 Derivation of Asset value of energy commodities from mean – reverting logistic Brownian motion with Jump diffusion model

We consider a logistic Brownian motion equation given by equation (1.1) as;

$$\frac{dX_t}{X_t(X^* - X_t)} = \alpha(\bar{X} - \ln X)dt + \sqrt{\upsilon_t}dW_t + dq \qquad (3.1)$$

The Left Hand Side of equation (3.1) is solved using Heave – side cover – up method to obtain a solution of the form; M. O. Opondo [9].

$$\frac{dX_t}{X_t(X^* - X_t)} = \frac{A}{X_t} + \frac{B}{X^* - X_t} \qquad (3.2)$$

To solve for A we equate the following $\frac{dX_t}{X_t(X^* - X_t)} = \frac{A}{X_t}$ which implies that $A = \frac{X_t dX_t}{X_t(X^* - X_t)}$. When we let $X_t = 0$. We obtain;

$$A = \frac{1}{X^*}dX_t \qquad (3.3)$$

To solve for B we equate the following $\frac{dX_t}{X_t(X^* - X_t)} = \frac{B}{X^* - X_t}$ which implies that $B = \frac{(X^* - X_t)dX_t}{X_t(X^* - X_t)}$; Letting $X^* = X_t$ we obtain the following;

$$B = \frac{1}{X^*}dX_t \qquad (3.4)$$

Substituting equation (3.3) and (3.4) into equation (3.2), we get the following;

$$\frac{dX_t}{X_t(X^* - X_t)} = \frac{\frac{1}{X^*}dX_t}{X_t} + \frac{\frac{1}{X^*}dX_t}{(X^* - X_t)} \qquad (3.5)$$

Substituting equation (3.5) onto (3.1) we obtain the following;

$$\frac{\frac{1}{X^*}dX_t}{X_t} + \frac{\frac{1}{X^*}dX_t}{(X^* - X_t)} = \alpha(\bar{X} - \ln X)dt + \sqrt{\upsilon_t}dW_t + dq \qquad (3.6)$$

When we integrate equation (3.6) with respect to dt, we get the following;

$$\frac{1}{X_t}\ln|X_t| + \frac{1}{X^*}\left(-\ln|X^* - X_t|_{t_0}^{t}\right) = \alpha(\bar{X} - \ln X)t|_{t_0}^{t} + \sqrt{\upsilon_t}dW_t + q(t) \qquad (3.7)$$

Equation (4.7) is simplified further into;

$$\frac{1}{X_t}\ln\left|\frac{X_t}{X^* - X_t}\right|_{t_0}^{t} = \alpha(\bar{X} - \ln X)t|_{t_0}^{t} + \sqrt{\upsilon_t}dW_t + q(t) \qquad (3.8)$$

Solving the limits by letting $X_{t_0} = X_0$, we get

$$\ln\left|\frac{X_t}{X^* - X_t}\right| - \ln\left|\frac{X_0}{X^* - X_0}\right| = \alpha(\bar{X} - \ln X)X^*(t - t_0) + X^*\sqrt{\upsilon_t}dW_t + X^*q(t) \qquad (3.9)$$

The Left Hand Side of equation (3.9) can be simplified as;

$$ln\left|\frac{X_t(X^*-X_0)}{(X^*-X_t)X_0}\right| = \alpha(\overline{X} - ln\,X)X^*(t-t_0) + X^*\sqrt{\upsilon_t}dW_t + X^*q(t) \quad (3.10)$$

Taking exponential on both sides of equation (3.10) we get the following;

$$\frac{X_t(X^*-X_0)}{(X^*-X_t)X_0} = exp\{\alpha(\overline{X} - ln\,X)X^*(t-t_0) + X^*\sqrt{\upsilon_t}dW_t + X^*q(t)\} \quad (3.11)$$

Solving for X_t from equation (3.11) we obtain the following

$$X_t = \frac{X^*X_0}{X_0 + (X^*-X_0)e^{-[\alpha(\overline{X}-ln\,X)X^*(t-t_0)+X^*\sqrt{\upsilon_t}dW_t+X^*q(t)]}} \quad (3.12)$$

Equation (3.12) is the price dynamic of mean – reverting logistic Brownian motion with jump diffusion process which can be used to analyze prices of energy commodities, in particular in this paper we look at prices of Kenya Power and Lighting Company in the next subsection,

3.2 Line Graph analysis of prices of energy at Kenya Power and Lighting Company from 2016 - 2020

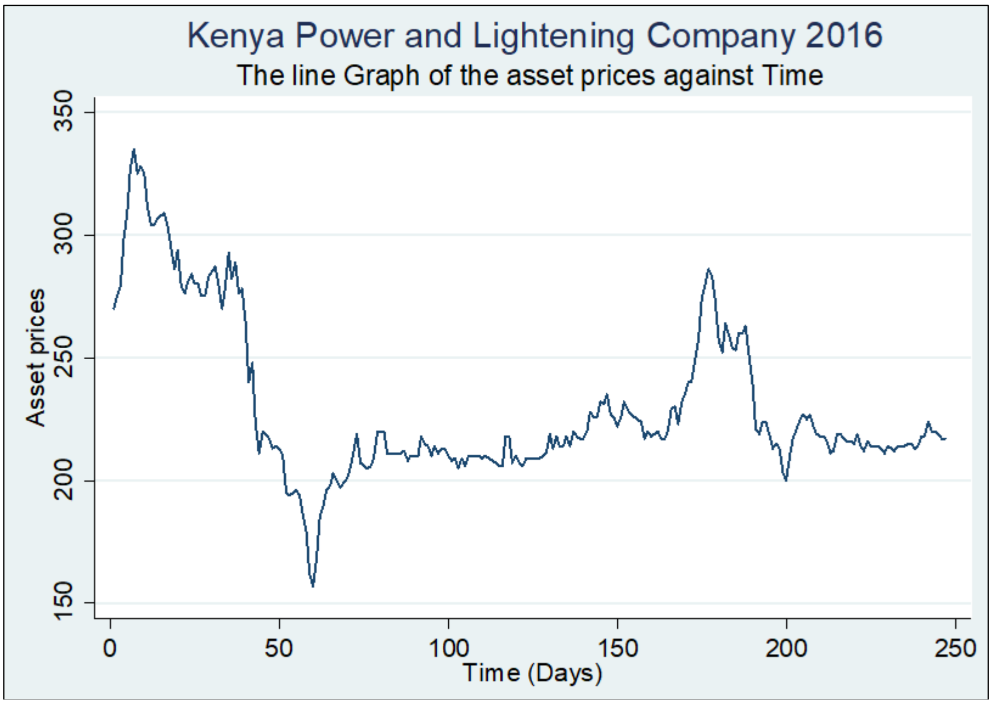

Fig 1: Kenta Power and Lightening Company 2016, the line Graph of the asset prices against Time

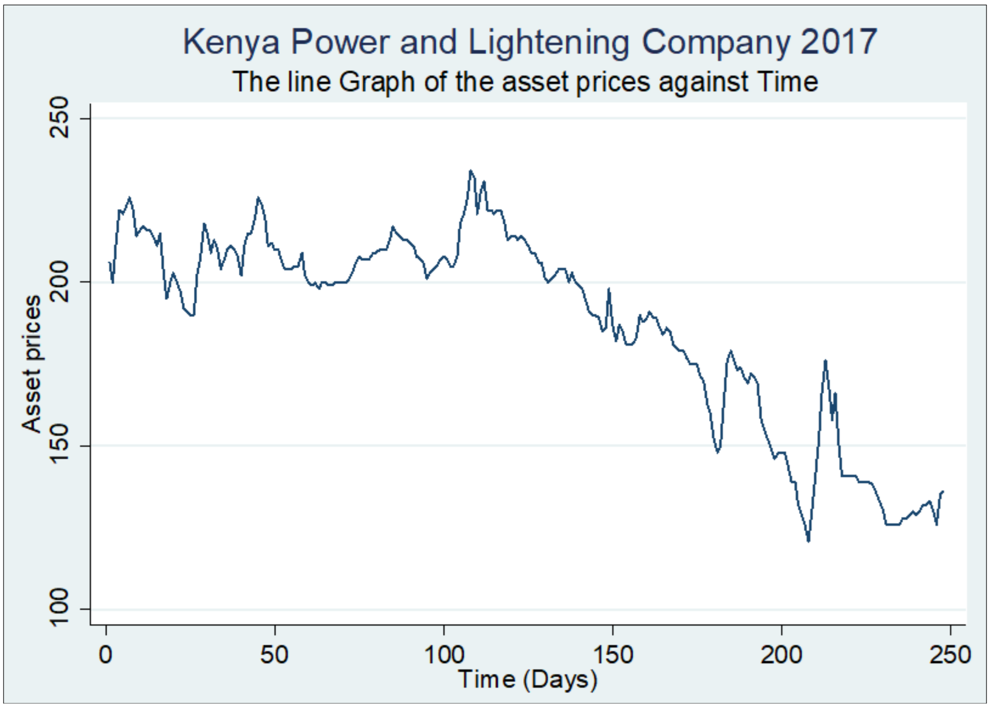

Fig 2: Kenta Power and Lightening Company 2017, the line Graph of the asset prices against Time

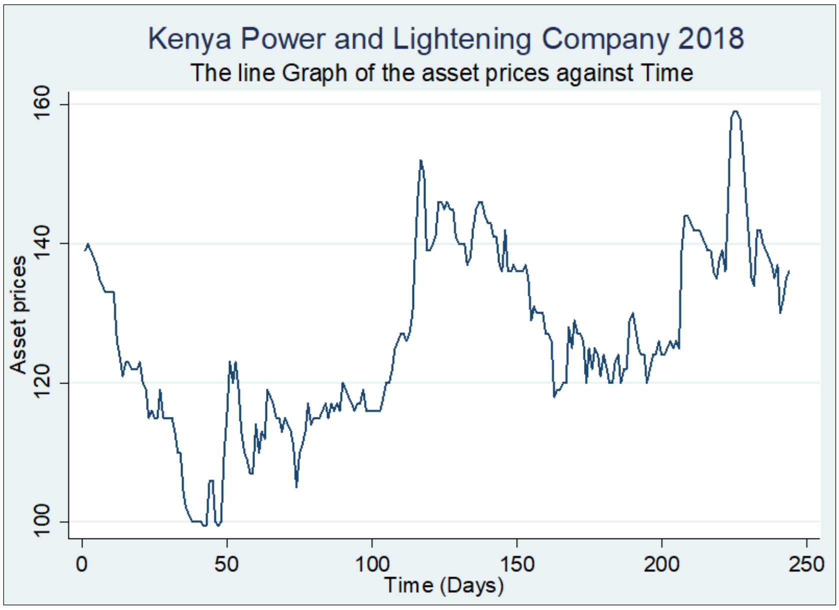

Fig 3: Kenta Power and Lightening Company 2018, the line Graph of the asset prices against Time

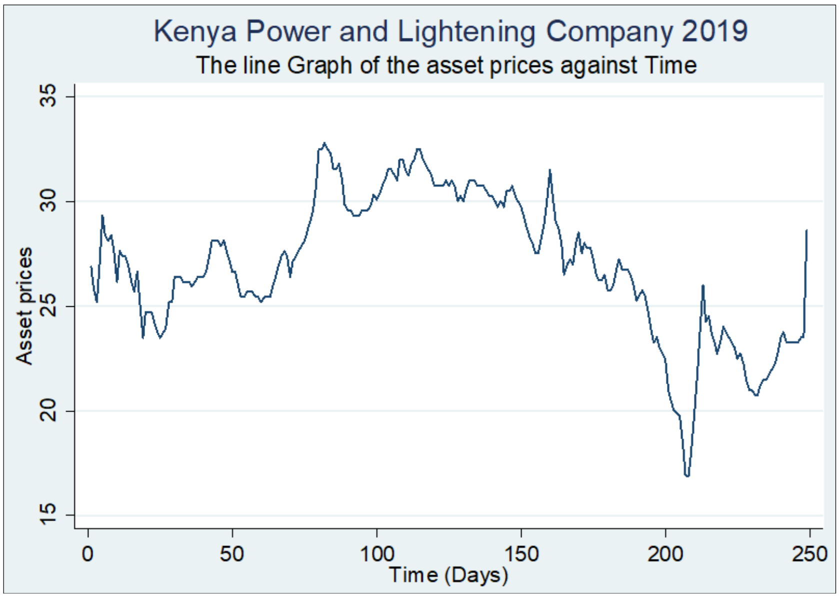

Fig 4: Kenta Power and Lightening Company 2019, the line Graph of the asset prices against Time

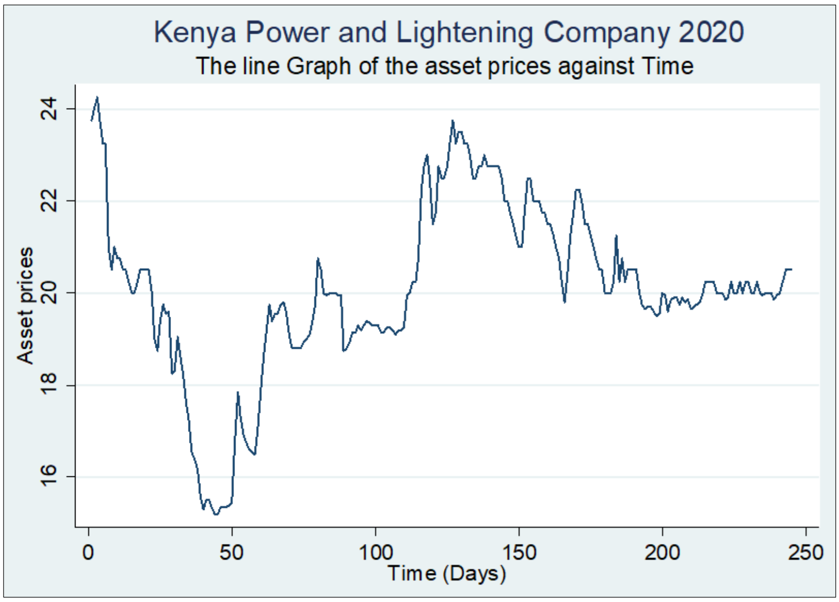

Fig 5: Kenta Power and Lightening Company 2020, the line Graph of the asset prices against Time

4. Conclusion

In this paper we have derived price dynamic of mean – reverting logistic Brownian motion with jump diffusion process which can be used to analyze prices of energy commodities. In particular we have presented line graph of prices of energy commodities from Kenya Power and Lighting Company.

5. References

1. Andanje Mulambula, Oduor DB, Kwach B. Derivation of Black - Scholes - Merton Logistic Brownian motion differential equation with Jump diffusion process. Int. J. Math. And Appl. 2019;7(3):85-93. ISSN 2347-1557
2. Andanje Mulambula DB Oduor, Kwach B. Volatility estimation using European Logistic Brownian motion with Jump diffusion process. Int. J. Math. And Appl. 2020;8(2):155-163. ISSN 2347-1557
3. Black F, Scholes N. The pricing and corporate liabilities, Journal of political economy. 1973;18:637-659.
4. Oduor DB, Omolo N, Ongati NB, Okelo, Silas N. Onyango. Estimation of market volatility: A case of Logistic Brownian Motion. International Journal of Marketing and Technology. 2012;2(1). ISSN: 2249-1058.
5. Heston S. A closed - form solution for options with stochastic volatility with application to bond and currency options; Review of Financial studies. 1993;6(2):327-343.
6. Hull CJ. Options, Futures and other derivatives, Joseph L. Rotman School of Management, University of Toronto 4th Edition, 2000.
7. Hull JC, White A. The pricing of options on assets with stochastic volatilities, Journal of finance. 1987;42:281-300.
8. Merton Robert C. Option pricing when underlying stock returns are discontinuous, Massachusetts Institutre of Technology. 1975;16(22):225-443.
9. Opondo MO, Oduor DB, Odundo F. Jump diffusion Logistic Brownian Motion with dividend yielding asset. Int. J. Math. And Appl. 2021;9(4):25-34. ISSN 2347-1557
10. Nyakinda, Joseph Otula. A logistic Non – linear Black – Scholes – Merton partial differential equation: European Option, International Journal of Research – Granthaalayah. 2018;6(6):480-487.
11. Nyakinda, Joseph Otula. Derivation of Non – linear Logistic Black – Scholes – Merton Partial differential equation, PhD Thesis, Jaramogi Oginga Odinga University of Science and Technology, 2011.
12. Oduor Brian D. Derivation of black Scholes equation using Heston's model with dividend yielding asset. International Journal of Statistics and Applied Mathematics. 2022;7(1):8-12. ISSN: 2456-1452.
13. Oduor Brian D. Formulating Black Scholes equation using a Jump diffusion Heston's model. International Journal of Statistics and Applied Mathematics. 20227(1):13-18. ISSN: 2456-1452.
14. Oduor Brian D. A combination of dividend and jump diffusion process on Heston model in deriving Black Scholes equation. International Journal of Statistics and Applied Mathematics. 2022;7(1):8-12. ISSN: 2456-1452.
15. Oduor DB, Onyango SN. A logistic Brownian motion with price of dividend yielding asset, International Journal of Research and Social Sciences. 2012;2(2).
16. Stein EM, Stein JC. Stock price distribution with stochastic volatility: An Analytic approach, Review of financial studies. 1991;4:727-752.