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A transfer function modelling of Nigerian current account (Net) and exchange rate

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Abstract
This study proposes a transfer function noise model for the Nigerian current account (net) using the US dollar exchange rate as the input variable. The sixty annually collected time series data used covering the period 1960-2019 were obtained from the National Bureau of Statistics. Both series were non-stationary and were transformed by differencing. The transformed series were confirmed stationary using an appropriate test, the augmented Dickey-Fuller test, ADF. The cross-correlation function method was used for the estimation of the transfer model while the Box-Jenkins autoregressive integrated moving average method was used to separately model the autocorrelated noise term. The estimated transfer function with a noise model was subjected to a diagnostic check. The calculated Q-statistic offered evidence for the adequacy of the estimated model. For parsimony, estimates for the model parameters were recursively obtained after factorization.

Keywords: Transfer function model, current account, exchange rate, time series, noise model

Introduction
The current account of a country is an economic variable instrumental in showcasing the economic performance of that country. It determines the country's balances of trade in goods, services as well as its net income and also relates the local economy of that nation and the economies of other nations. This means that the current account exposes the financial and economic health of a nation thereby prescribing how trade activities influence the country. The International Monetary Fund in 2009 stated that the current account together with the capital and financial accounts constitutes the major components of the Balance of Payments (BoP). Hence, it is a significant feature of economic decisions.

Ismaili-Muharremi (2015) [9] mentioned the lack of research in the subject of current account especially its deficit and hence analysed the ability to sustain the current account of a country's economy using data from six different Balkan countries. Their study identified domestic production, increased quantity of exports, and constant flow of foreign direct investments to be the three impellers of a country's current account in those six countries. Oshota *et al* (2015)[15] identified recurrent deficits as major characteristics of the current account balance in West Africa. The recurrent deficits lead to other factors in accounting for the imbalances in the Balance of Payments. Empirical study has also suggested that overrated real exchange rates among other factors, induces the event and continuance of deficits in the current account in most countries in West Africa. This signals a need to investigate the interrelation between the current account and the exchange rate.

The Nigerian current account has records of inequality and paucity with large external debts burdens giving rise to a pauperized economy. This is not unrelated to its high dependence on importation. The fact that the Nigerian economy depends mainly on the exportation of crude oil has a crucial impact on the other macroeconomic variables as studied by Henry (2019) [7] and Ogundipe *et al* (2014) [12].

Statistical models such as time series models, use patterns in the data to establish what produces and/or impacts the patterns as well as the variations involved in the data. It is also import to study these variations jointly in order to help reveal underlying relationships among

time series (Victor-Edema & Essi, 2016) [19]. One of such models is the transfer function model introduced by Box and Jenkins (1976) [4]. The model illustratesthe relationship between an output series Y and one or more inputseries X. This helps in explaining the correlated frame of the time series. Transfer function models are broadly applied in different areas. For instance, Otok (2009) [16] estimated a transfer function model for the Indonesian rainfall index and compared the precision of the forecast from the transfer function model with other time series models. The resultsrevealed that thetransfer function model performed better. Huang and Wu (2014) [7] showed that the transfer function model was a better model for predicting students' academic achievement of college entrance when compared with the predictions from the ARIMA model. In another work Okiy *et al* (2015) [11] estimated a transfer function model for the electricity price forecast, Camargo *et al*. (2010) [4] investigated the patterns of the Brazilian inflationary system, Moroke (2015) [11] studied the relationship between investments and savings in South Africa, Iwok (2015) [10] estimated a transfer function noise model for the naira exchange rate to US dollar and Swiss franc and Alem *et al.* (2018) [3] modelled the effect of farm management and socio-economic factors on crop yield in Norway.
Some studies have been done in time past on the Nigerian current account. For instance, Adebayo *et al.* (2018) [1] investigated the relationship between the current account and fluctuations in oil price, Oseni and Onakoya (2013) [14] empirically analysed fiscal policy shocks and the Nigerian current account dynamics. Some other works have also applied theoretical models to discuss the factors that affect the current account. One of such works was done by Adedeji (2001) [2] who presented a model for the determination of the current account based on the permanent income hypothesis. But no known work has been dedicated to investigating the relationship between the Nigerian current account (net) and the exchange rate in dollars using the transfer function model.

Materials and Method

2.1 Data

This study uses two sets of economic data. The annually collected data of the Nigerian current account (net) which is the output series and the monthly collected United States dollar exchange rates which were converted to annual data which is the input series. Both series were sourced from the Nigeria Bureau of Statistics and covered the period 1960 – 2019. The choice of the period was informed by the need to satisfy the normality assumption. Both time series data were non-stationary. The input and output series are designated as ($\mathrm{X_t}$ and $\mathrm{Y_t}$) respectively.

2.2 The Transfer Function Noise Model.

The transfer function model addresses the limitation of the univariate ARMA because it can make room for more than one input variable. This study employs a single input – exchange rate to estimate a forecasting model for the output – Nigerian current account. Let X_t *and* Y_t be the input and output series respectively. In reality, the two variables do not always share an accurate relationship, it is dependent upon noise. The model is stated as;

$$Y_t = \mu + \sum_{j=0}^{\infty} v_j X_{t-j} + n_t \ (-\infty < t < \infty) \tag{1}$$

where
μ = constant
$\sum_{j=0}^{\infty} v_j$ = transfer function filter
$v_j = 0$ for all $0 \leq j \leq k - 1$, k is the delay time before the input starts to affect the output, also referred to as the impulse response weights. According to Wei (2009), the transfer function is stable only if the impulse weights are completely summable. That is $\sum |v| < \infty$
n_t = noise term. The noise term must be uncorrelated with the input variable X_t.
The transfer function is sometimes represented as a rationale to palliate the challenges associated with the finite input and output having an infinite number of coefficients. Thus;

$$\mathrm{v(B)} = \frac{\omega(B)B^b}{\delta(B)B^b} \tag{2}$$

where, $\omega(B)$ *and* $\delta(B)$ = polynomials taking B as the variable where b is a parameter that estimates the delay between the variables. The ratio $\frac{\omega(B)}{\delta(B)}$ is called the transfer function of the system.

$$\omega(B)B^b = v(B)\delta(B)$$

Again, the transfer function model can be written as

$$Y_t = \frac{w(B)}{\delta(B)} X_{t-b} \tag{3}$$

2.3 Stationarity

The typical unit root Augmented Dickey-Fuller test performed using the appropriate lag length to ensure the lag length is adequate and does not exact on the power test.

$$\tau_{ADF} = \frac{\phi_1 - 1}{SE(\hat{\phi}_1)} \tag{4}$$

where t_{ϕ_1-1} is called test statistic of $\phi_1 - 1$, and $SE(\hat{\phi}_1)$ depicts the standard error of $\phi_1 - 1$. The null hypothesis is rejected if the test statistic in equation () is less than the standard critical value at a certain significance level, or the value of the corresponding probability exceeds the level of significance.
A transformed transfer function noise model with the same order of differencing is given as;

$$\nabla Y_t = V(B)\nabla X_t + n_t \quad (5)$$

where $v(B) = (v_0 + v_1 B + v_2 B^2 + \cdots)$ is called the transfer weight or impulse response function and n_t is the noise term.

2.4 Cross-correlation function
The cross-correlation function CCF identifies the standard of association between the two variables in this study, a bivariate stochastic process. The CCF is given by its estimator;

$$\gamma_{xy}(k) = \frac{Cov_{xy}(k)}{S_x S_y} \qquad \text{for k = 0, 2, 3, …} \quad (6)$$

The transfer function weights are estimated from the equation'

$$\hat{v}_k = \frac{r_{\alpha\beta}(k) S_\beta}{S_\alpha} \qquad \text{k=0,1,2,3} \quad (7)$$

2.4 Model identification
Preliminary identification of the transfer function model for the transformed series is done using the cross-correlation function. Since Xt and Yt became stationary after first difference, the transformed series is $x_t = \nabla^d X_t$ and $Y_t = \nabla^d Y_t$.

2.4.1 Pre-whitening
This involves fitting an ARIMA(p,d,q) model to the input variable X_t that produces a white noised residual a white noise yet maintains the relationship between X_t and Y_t. yielding

$$\alpha_t = \phi_X^{-1}(B)\theta_x(B)X_t \quad (8)$$

where α_t are residuals from the estimated model and are white noise. $\alpha_t$filters the output series yielding

$$\beta_t = \phi_X^{-1}(B)\theta_x(B)Y_t \quad (9)$$

2.5 Model Estimation
The identified transfer function weights are estimated from the cros- correlation function;

$$v_k = \frac{r_{\alpha\beta}(k)\sigma_\beta^2}{\sigma_\alpha^2}, k = 0, \pm 1, \pm 2, \ldots \quad (10)$$

where, v_k is the transfer function weight. This preclusive estimates for the transfer function weights is not sufficient statistically but it can only hint on good operators $\delta(B)$and $w(B)$ in the main model in equation 3.

2.6 The Noise Model
The noise model is estimated by an ARIMA(p,d,q) process.

$$n_t = \frac{\theta(B)}{\varphi(B)} e_t \quad (11)$$

where e_t is white noise. Therefore, the transfer function noise model is stated as;

$$Y_t = \delta^{-1}(B)w(B)X_{t-b} + \varphi^{-1}(B)\theta(B)e_t \quad (12)$$

The parameters and the residuals are estimated using the Box and Jenkins (1970) three-stage conditional likelihood estimation technique.

2.7 Diagnostic Checks
Diagnostic checks help to validate the assumptions of an estimated model and check the adequacy of the evaluated model. These are done by using standard tests. The test for autocorrelation and cross-correlation for the estimated model are done using the modified McLead Q-test. This test statistic tests the null that the identified transfer function model is adequate is given as;

$$\hat{Q} = m(m+2)\sum_{k=1}^{K}(m-k)^{-1} r_{e_t e_t}^2 \quad (13)$$

where Q is expressed like the case of ARIMA model. The number of parameters is shown in m, k is the number of lags and $r_{e_t e_t}^2$ is lagged cross correlation of the residuals $\hat{e}_t$.

Results and Discussion

The bivariate plot of the input and output series is displayed in figure 1. The correlation coefficient of 0.497 indicates a positive correlation between Nigerian current account and exchange rate in dollars. But there is no significant correlation between the transformed input series ∇X_t and the transformed output series ∇Y_t (correlation =0.120, p-value = 0.354). Visible inspection of the time series plots of both series show non-stationarity at level, but stationarity after first difference with zero mean and constant variance. The ADF unit root tests on the transformed series further verified the claim of stationarity at first difference. See table.

Figures 2 and 4 present the ACFs and the PACFs of the series. It is observable from the charts that the single-differenced series shows the two series are integrated with order one.

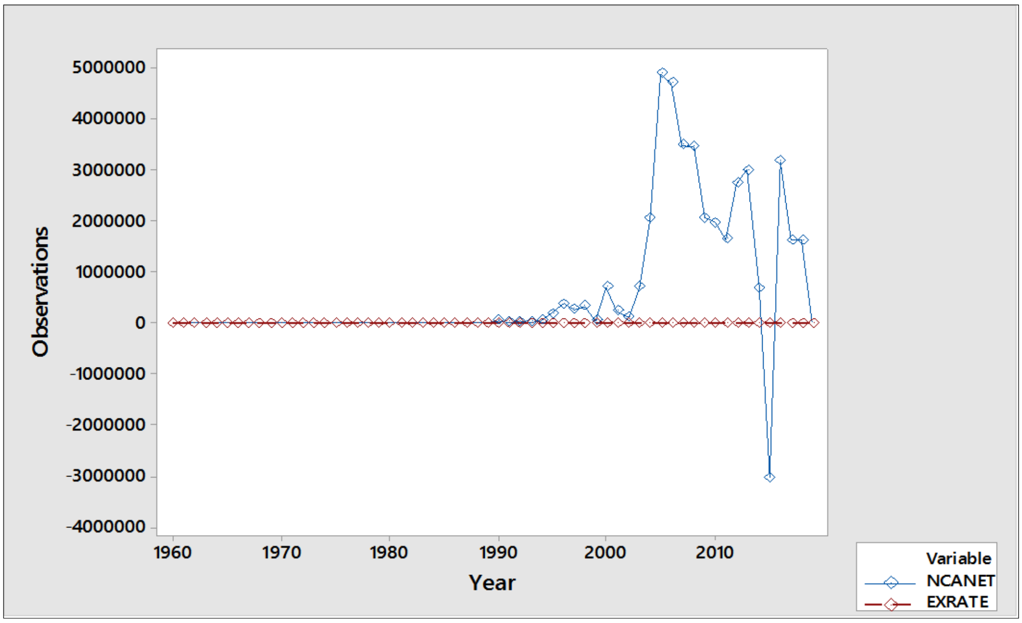
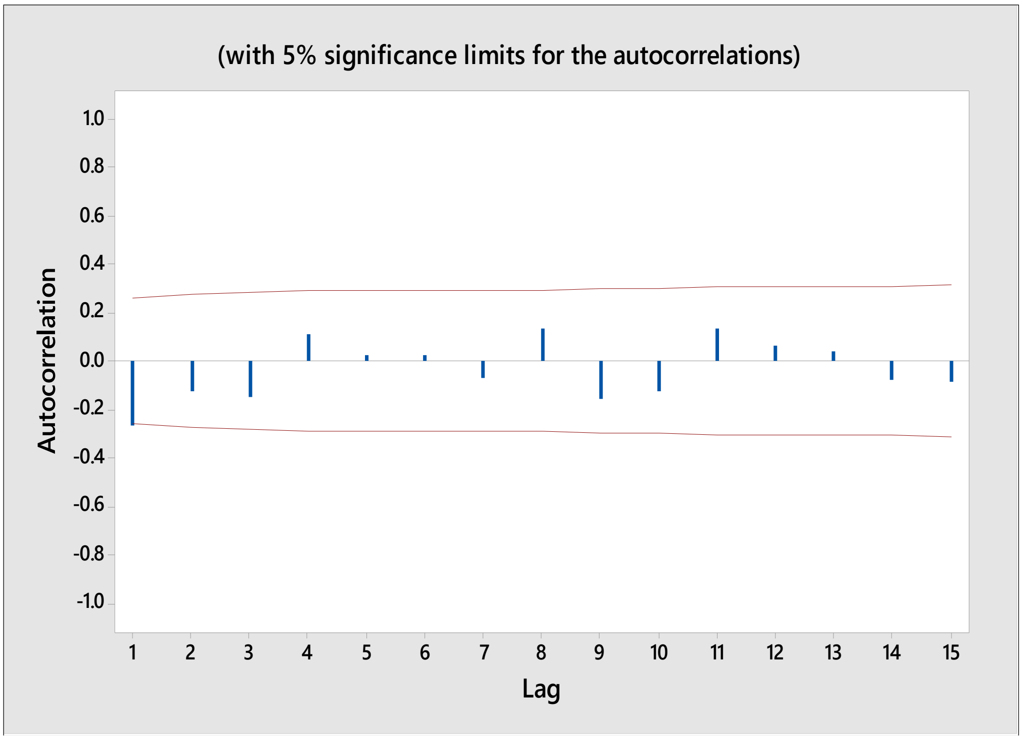


Fig 1: Bivariate Plot of X_t and Y_t



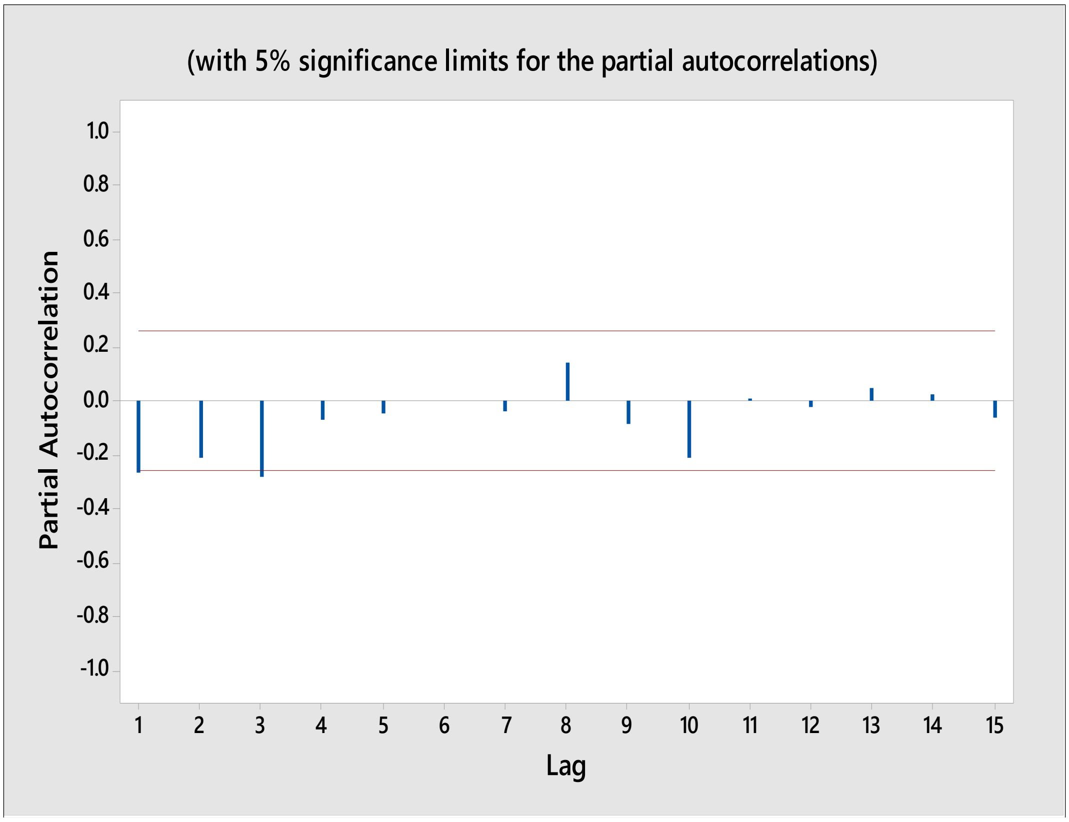


Fig 2: Plot of ACF and PACF for Output Time Series ∇Y_t

3.1 Model identification for input Series Xt

An ARIMA model was fitted to the input series using the Box-Jenkins iterative method. The ACF and the PACF suggested AR (1) or MA (1) or ARMA (1,1). The best ARMA model was selected after fitting different suggestive models using the Akaike inspection criterion (AIC). See table 1. The identified suitable model is AR (1) with first difference, that is ARIMA (1,1,0)].

$\alpha_t = (1 - \varphi_1 B)(1 - B)X_t$

Thus, the ARIMA model for the exchange rate series is

$$\alpha_t = (1 - 0.4411B)(1 - B)X_t \tag{14}$$

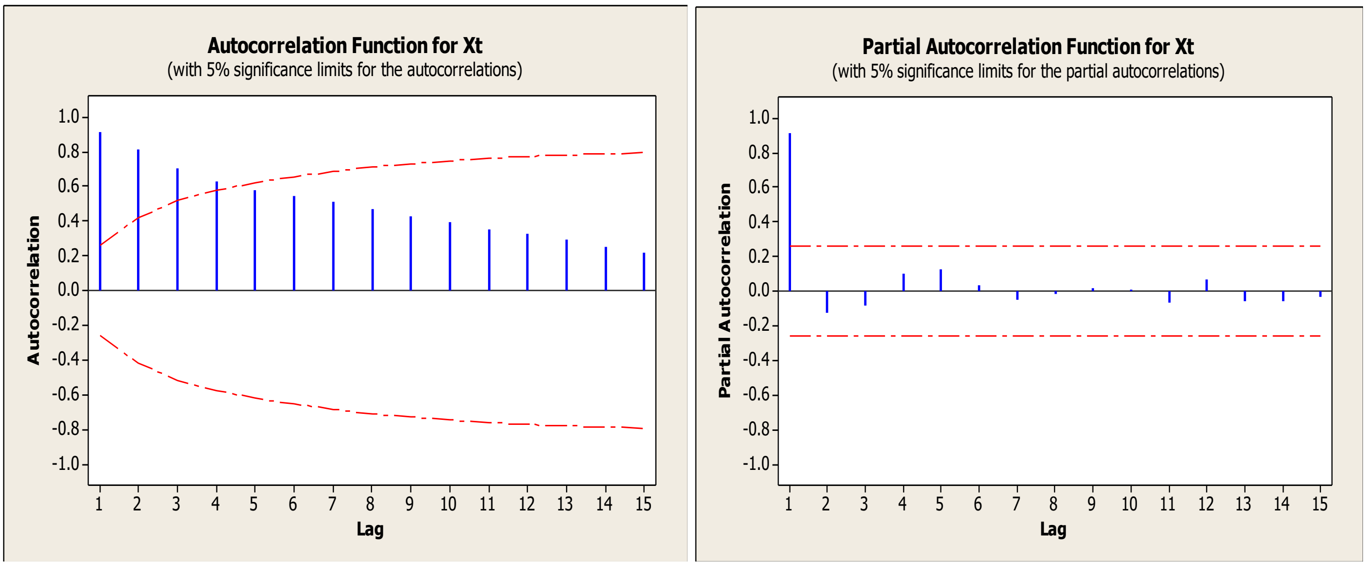


Fig 3: Plot of ACF and PACF for Input time series X_t

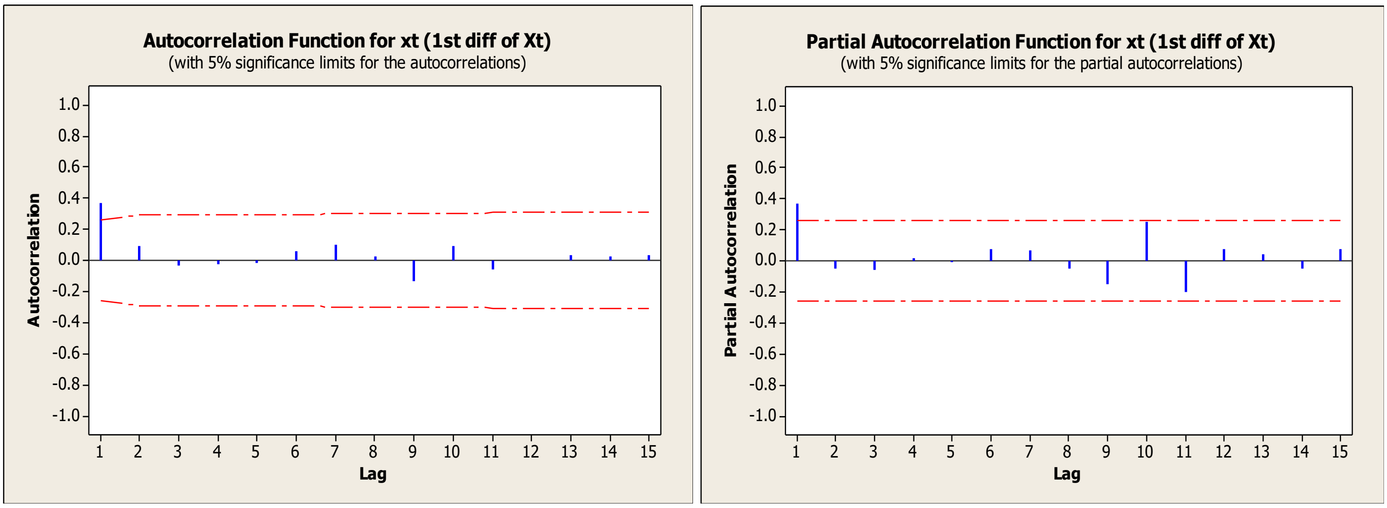


Fig 4: Plot of ACF and PACF for Input time series ∇X_t

Table 1: Arima Model Estimates for the Input Series

Models	AR(p) Estimates	MA(q) Estimates	Modified Box-Pierce (Ljung-Box) Chi-Square statistic				AIC
Arma Models	φ_1	θ_1	k=12	k=24	k=36	k=48	
ARMA (1,0)	0.4411(0.000***)		11.1 (11)	22.8 (23)	23.5 (35)	24.3 (47)	552.68
ARMA (0,1)		-0.3734 (0.004***)	11.0 (11)	21.8 (23)	23.6 (35)	26.1 (47)	554.98
ARMA (1,1)	0.4541 (0.095*)	0.0161 (0.957)	11.0 (10)	22.8 (22)	23.5 (34)	24.3 (46)	554.67

Footnote: ***-sig. at 1%, **-sig. at 5%,*-sig. at 10%; m=number of parameters and p-values in parenthesis.

3.2 Pre-Whitening of the Input X_t and Output Y_t

By expanding the ARIMA model and solving for the residuals

$$\hat{\alpha}_t = X_t - (\varphi_1 + 1)X_{t-1} + \varphi_1 X_{t-2} \tag{15}$$

For the input series the equation becomes

$$\hat{\alpha}_t = X_t - 1.4411X_{t-1} + 0.4411X_{t-2}$$

And for the output time series Y_t the pre-whitened output $\hat{\beta}_t$

$$\hat{\beta}_t = Y_t - 1.4411Y_{t-1} + 0.4411Y_{t-2}$$

3.3 Cross Correlation Function (CCF) for the Prewhitened Series

The pre-whitened series $\hat{\alpha}_t$ and $\hat{\beta}_t$ were cross-correlated at a set significant boundary of $\pm\frac{2}{\sqrt{n}} = \pm\frac{2}{\sqrt{60}} = \pm 0.26$ where n is the total number of observations. The chart of the CCF is in figure gives the non zero transfer weight at lag 2 (-0.393) making the delay time b=2, meaning the transfer weights lag 0 and lag 1 are statistically insignificant. Furthermore, no significant spike in CCF after lag 2 up to lag 15 as shown in table 2.

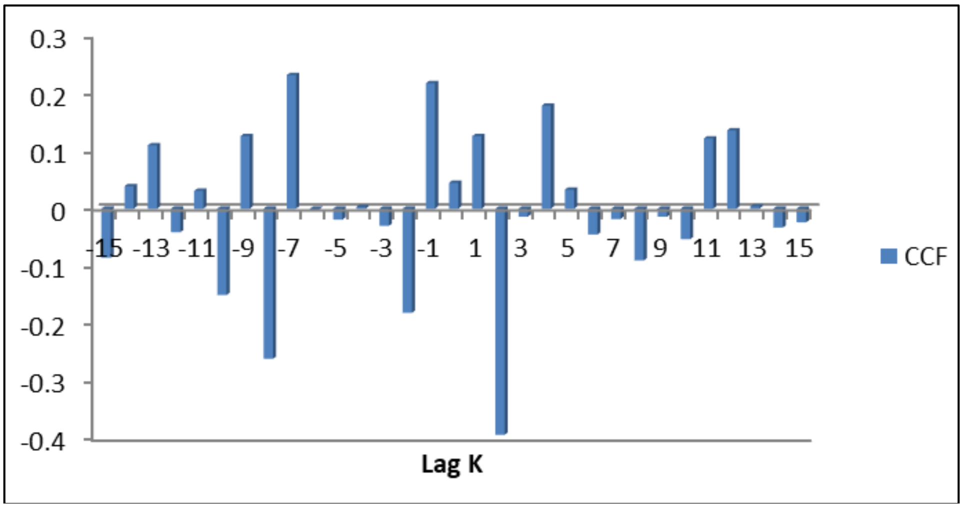


Fig 5: Cross-Correction Function (CCF) between $\hat{\alpha}_t$ and $\hat{\beta}_t$

Table 2: Cross-Correlation Function and Transfer Function Weights

K	CCF	weight v_k	Transfer Impulse Function $\hat{v}_k$
0	0.05	4577.63	0
1	0.13	12817.35	0
2	-0.39	-39977.94	-39977.90
3	-0.01	-1424.15	0
4	0.18	18208.78	0
5	0.03	3356.93	0

3.4 Identification and Estimation of the Transfer Function Weights

Expanding the polynomials in equation 3 becomes

$$(1-\delta_1 B-\delta_2 B^2-\cdots\delta_r B^r)(v_0+v_1 B+v_2 B^2+v_3 B^3+\cdots)=(w_0+w_1 B+w_2 B^2+\cdots+w_s B^s)B^b \quad (16)$$

Putting b=2, r=1, and s=0 into equation 16, expand and take coefficient gives

$$(1-\delta_1 B)(v_0+v_1 B+v_2 B^2+v_3 B^3+\ldots)=w_0 B^2$$

$$B^2\colon v_2=w_0$$

$$B^3\colon \delta_1 v_2+v_3=0 \Rightarrow \delta_1=0$$

Then substituting $w_0$and $\delta_1$into Equation 3 gives

$$Y_t=w_0 X_{t-b} \quad (17)$$

Again, solving for equation 16 when b=2, r=1 and s=1 and taking coefficient with B backward shift

$$(1-\delta_1 B)(v_0+v_1 B+v_2 B^2+v_3 B^3+\ldots)=(w_0-w_1 B)B^2$$

$$B^2\colon v_2-\delta_1 v_1=w_0, w_0=v_2$$
$$B^3\colon v_3-\delta_1 v_2=w_1$$
$$B^3\colon v_4-\delta_1 v_3=0$$
$$\delta_1=\frac{v_4}{v_3}=0$$

therefore $\delta_1=0$ evidences $w_1=0$
putting w_0, w_1 and δ_1 into equation 3 gives

$$Y_t=w_0 X_{t-b}$$

Finally, solving the same for b=2, r=2, and s=1, and substituting the parameters gives

$$(1-\delta_1 B-\delta_2 B^2)(v_0+v_1 B+v_2 B^2+v_3 B^3+\ldots)=w_0 B^2$$
$$B^2\colon v_2-\delta_1 v_1-\delta_2 v_0=w_0, w_0=v_2$$
$$B^3\colon v_3-\delta_1 v_2-\delta_2 v_1=0, -\delta_1 v_2=0, \therefore \delta_1=0$$
$$B^3\colon v_4-\delta_1 v_3-\delta_2 v_2=0 \Rightarrow -\delta_2 v_2=0, \therefore \delta_2=0$$

putting w_0, δ_1 and δ_2 into Equation 3 gives

$$Y_t=w_0 X_{t-b} \quad (18)$$

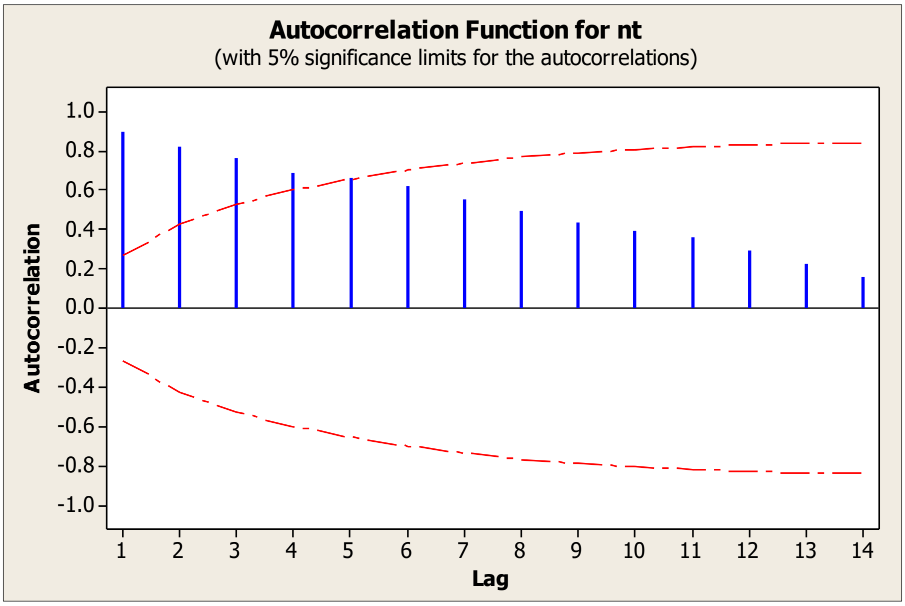
This shows that equation 17 is the estimated transfer function model for the output series because it is seen to be true for b=2 when r is 1 and s is 0, r and s are both 1, and when r is 2 while r is 2 and s is 1. Therefore, the identified transfer model for the output series Y_t with estimated w_0 as -39977.93 and b=2 is given as,

$$Y_t=-39977.93 X_{t-2}$$

3.5 Estimation of the Noise term

The noise model is estimated from ARIMA. The plots of ACF and PACF of Ut are given in figure 5 below

$$n_t=\nabla Y_t-y_t$$



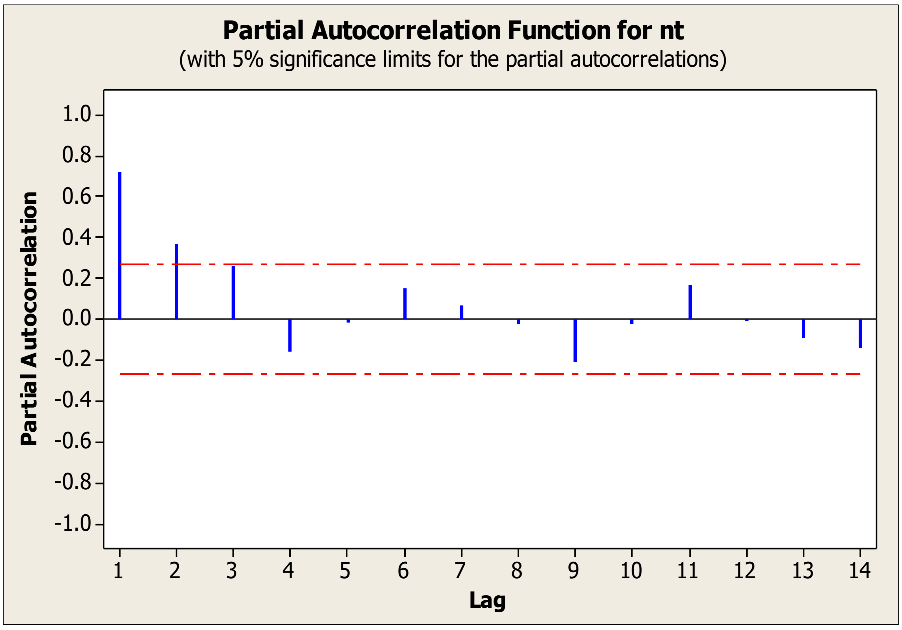


Fig 6: ACF and PACF for the Additive Noise n_t

The plots hint an autoregressive process, AR(2) to be precise. The ordinary least square estimates for AR(2) is

$$e_t = \varphi_1 n_{t-1} - \varphi_2 n - n_t$$

$$n_t = \frac{e_t}{1-\varphi_1 B-\varphi_2 B^2} \tag{19}$$

where φ_1 = 0.469, φ_2= 0.535

$$n_t = \frac{e_t}{1-0.469B-0.535B^2} \tag{20}$$

thus, a suggestive identified transfer function noise model is given as

$$\nabla Y_t = w_0 X_{t-2} + \frac{e_t}{1-\varphi_1 B-\varphi_2 B^2} \tag{21}$$

Therefore, the transfer function model with added noise is

$$Y_t = Y_{t-1} + w_0 X_{t-2} + \frac{e_t}{1-\varphi_1 B-\varphi_2 B^2} \tag{22}$$

$$Y_t = Y_{t-1} - 39977.93 X_{t-2} + \frac{e_t}{1-0.469B-0.535B^2} \tag{23}$$

3.6 Estimation of the Transfer Function Noise Model

The estimation was done by applying the least square method of estimation to the first factorized identified model. This was done in order to attain parameter parsimony. From the equation (22) we write;

$$(1-B)Y_t = w_0 X_{t-2} + \frac{e_t}{1-\varphi_1 B - \varphi_2 B^2}$$

$$(1-B)(1-\varphi_1 B - \varphi_2 B^2)Y_t = (1-\varphi_1 B - \varphi_2 B^2)w_0 X_{t-2} + e_t$$

$$Y_t = (1+\varphi_1)Y_{t-1} + (\varphi_1 - \varphi_2)Y_{t-2} - \varphi_2 Y_{t-3} + w_0 X_{t-2} - w_0\varphi_1 X_{t-3} - w_0\varphi_2 X_{t-4} + e_t$$

Substituting the values of the parameters give the estimated transfer function noise model for the Nigerian current account as;

$$Y_t = 1.469Y_{t-1} - 0.066Y_{t-2} - 0.535Y_{t-3} - 4.0\times10^4 X_{t-2} + 1.87\times10^4 X_{t-3} + 2.14\times10^4 X_{t-4} + e_t$$

3.7 Diagnostic Check

The adequacy of the estimated transfer function noise model in equation 22 was tested using the test statistic in equation 13. Given that m = n – p – q = 57 – 2 – 0 = 55, and K = 20, and $\sum_{k=1}^{20}(m-k)^{-1}r_{e_te_t}^2$= 0.0057. With degree of freedom of K – p – q = 20 – 2 – 0 = 18($\chi_{0.05}^2(18)$ = 9.39), the test gave no grounds for model inadequacy.

4. Conclusion

This study demonstrated the application of the transfer function noise model to the Nigerian current account (net) by using exchange rates in dollars as the input series. Sixty observations covering the period 1960-2019 were collected for each set of the time series and used for the data analysis. Both time series were non-stationary at level this necessitated transformation which was done by differencing once. Stationarity after the first difference was confirmed by the results of the augmented Dickey-Fuller unit root tests. The cross-correlation function method to identify a transfer function model proposed by Box and Jenkins was applied. ARIMA model was estimated for the noise term. Finally, the combined transfer function model with noise term for the Nigerian current account (net) was estimated, diagnostic checks were performed on the estimated model and found to be adequate for further application.

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