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New families and invariance under E-cordial labeling

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Abstract
In this paper we discuss new families of E-cordial labeling. These are FL(Sn), $(S_4)^{(k)}$, Crown of shel, $(Sn)^{(k)}$, $(K_{1,n}:K_{1,n})$, $(FL(C_4))^{(k)}$, E-cordial invariance of $(FL(C_3))^{(k)}$

Keywords: E-cordial, labeling, shel, flag grap, h invariance

1. Introduction
In 1997 Yilmaz and Cahit [5] introduced a weaker version of edge-graceful called E-cordial. labeling. Let G be a graph with vertex set V and edge set E and let f a function from E to {0, 1}. Define f on V by f(v) =Σf(uv) (mod 2).
The function f is called an E-cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1. A graph that admits an E-cordial labeling is called E-cordial.
The graphs considered are finite, undirected, simple and connected. For terminology and definations we refer Harary [4], Dynamic survey of graph labelling [3]. Further by vf(0, 1)=(a, b) we mean number of vertices with label 0 are a in number and that with label 1 are b in number. Similar convention is followed for edge numbers $e_f(0, 1) = (x, y)$.

3. Definitions
3.1. Shel Sn is obtained from Cn by taking n-3 chords starting with the same vertex on Sn say v and ending on the vertices of Cn that are not adjacent to v.
3.2. FL(G) is a flag graph of G and is obtained by attaching a copy of $k_2$to a suitable vertex of G. If This vertex is changed then the graph may differ structurally. For FL(Cn) and FL(Kn) which vertex is used to attach K_2 does not matters. For Flag of Sn we attach K2 at apex of Sn.
3.3. Antena graph Consider a G = (p, q) graph. At each of it's vertex attach a path of length m. then we get a antenna graph antenna (G, m). If we attach K antennas of different length at each vertex of G then it is k-antena(G).

4. Results proved
4.1. Theorem: Fl(Sn) is E-cordial iff n is even number.
Proof: Ordinary names given to vertices are that u_1 is the apex vertex and main cycle Cn of Sn is given by $(u_1, e_1, u_2, e_2,.. e_n, u_1)$. The chords are given by $ci=(u_1u_j)$,i = j-2 and j = 3, 4,.., n-2. Define a function f;E→{0,1}by f(ei)=0 for i is a even number and f(ei)= 1 for i>1and is an odd number. f(e1)=0. The chords n-3 in number are labeled as f(ci) =1 for i is an odd number and f(ei) = 0 for i is an even number. The final vertex numbers distribution we get is $v_f(0, 1)=(x+1, x)$ for p≡1(mod 4) and $v_f(0, 1)= (x, x+1)$ for p≡3(mod 4). and edge numbers are ef(0, 1) = (n-1, n-1).

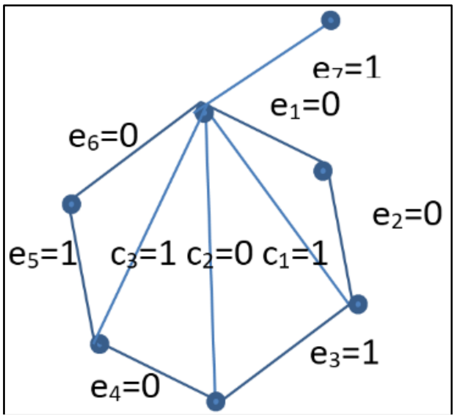

Fig 1: FL(S_6): E-cordial labeling

3.2. Theorem: One point union of k copies of S_4 i.e. $(S_4)^{(k)}$ is E- cordial

Proof: We use different types of labelings Type A, B, C, and D. We use different types of labelings that are explained in the Fig 2.

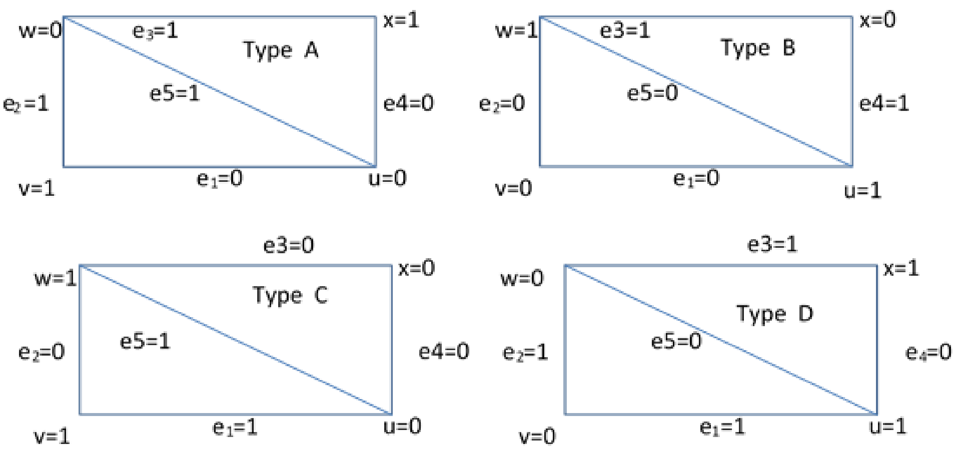

Fig 2: Four types of labeled copies of S_4

Table 1: explains labeling of $(S_4)^{(k)}$

k	Type Used	$v_f(0)$	$v_f(1)$	$e_f(0)$	$e_f(1)$	Comm-on vertex.
1	A	2	2	2	3	0
2	A+C	3	4	5	5	0
3*	A+C+B	4	6	8	7	1
4	A+C+B+D	7	6	10	10	0
4X	x times A, C, B, D	6X+1	6X	10X	10X	0
4X+1	x times A, C, B, D +A	6X+2	6X+2	10X+2	10X+3	0
4X+2	x times A, C, B, D +A+C	6X+3	6X+4	10X+5	10X+5	0
4X+3*	x times A, C, B, D +A+C+B	6x+6	6x+4	10x+8	10x+7	1

The table 1 explains the construction of G from using different types (A, B, C, D) of labels and resultant vertex and edge distribution. Note that * in the first column indicate that for that value of k the graph is not E-cordial. The note made by Yilmaz and Cahit is also observed here, that for number of vertices congruent to 2(mod 4), the graph is not E- cordial.

3.3 Theorem: $(K_{1,n}: K_{1,n})$ is E-cordial.

Proof: Refer the diagram.

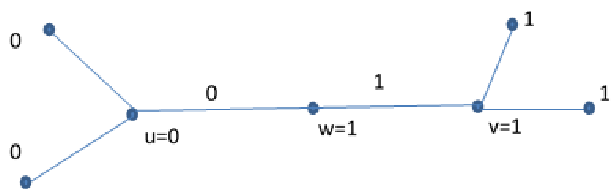

Fig 3: $(K_{1,3}:K_{1,3})$ is E-cordial

Label all the pendent vertices incident with u as 1 and that at v as 0.edge (uw) is labeled as 0 and edge (wv) as 1.
The label numbers are given by:
for n is even
$v_f(0)$=n+1 $v_f(1)$= n on vertices.
$e_f(0)$=n = $e_f(1)$ on edges.
For n is odd,
$v_f(0)$ = n, $v_f(1)$= n+1 on vertices.
$e_f(0)$=n = $e_f(1)$ on edges. Thus G is E-cordial.

3.4. Theorem: All three structures of G = $(FL(C_3))^{(k)}$ i.e. One point union of k- copies of FL(C_3) is E-cordial except for 3k≡1(mod 4)

Proof: There can be three distinct points on FL(C_3)which are used in three cases to obtain one point union. These points on FL(C_3) are i) pendent vertex ii) degree 2 vertex iii) degree 3 vertex. We discuss three cases separately.
Case 1: The common point is pendent vertex u.

The union point is taken on pendent vertex of $FL(C_3)$.The i^{th} copy of $FL(C_3)$ is defined as ($u\ e_{i1}\ u_{i2}\ e_{i2}\ u_{i3}\ e_{i3}\ u_{i4}\ e_{i4}\ u_{i2}$) i =1,2, k. We use two type of labeling namely type A and Type B to design G. Fig. 4 gives details.

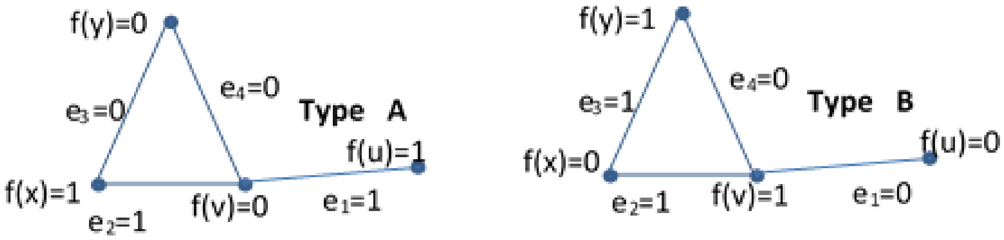

Fig 4: Labeled copy of Type A and Type B. These are building blocks of G =

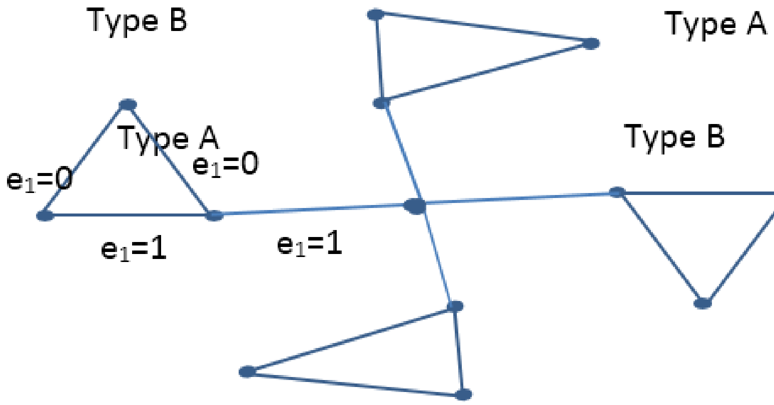

Fig 5: Pendent vertex is joined to obtain the structure

The copies of $FL(C_3)$ in G are numbered as 1, 2, k.

To obtain a labeled copy of G we start with (for k=1) Type A followed by Type B (For k =2). For K=3 we use Type A two times and type B one time. For k=4 we use three times Type A and one time Type B. After k=4 for every k^{th} copy is labeled depending on k≡1, 2 (mod 4) Type B is used and for k ≡ 0, 1 (mod 4) we use Type A label.

Table 2: Type 1 structure. Note that * in the first column indicate that for that value of k the graph is not E-cordial.

k	Type of label	f(u). It is label of common vertex	$v_f(0)$	$v_f(1)$	$e_f(0)=e_f(1)$
1	A	1	2	2	2k
2	A+B	1	3	4	2k
3*	A+B+A	0	6	4	2k
4	A+B+A+B	1	7	6	2k

The Edge distribution is always $e_f(0)=2k=e_f(1)$ for every k.

The vertex distribution we have, (for k>4)

For k ≡ 0(mod 4) we have $v_f(0)= 3k/2+1, v_f(1)=v_f(0)-1$

For k ≡ 1(mod 4) $v_f(0)=(3k+1)/2$; $v_f(1)=v_f(0)$

For k ≡ 2(mod 4) $v_f(0)=3k/2$; $v_f(1)=v_f(0)+1$

For k ≡ 3(mod 4) $v_f(0)=(3k+3)/2$; $v_f(1)=v_f(0)-2$

Thus except for k ≡3(mod 4) we have G is E- cordial.

This is equivalent to observation if n≡2 (mod 4) the graph is not E-cordial. Where n is number of vertices on graph and is given by 3k+1.

Case 2: The one point union is taken at point v whose label in Type A is 0 and that in type B is1. These are opposite to label of pendent vertex in respective type. Therefore we Label the graph Starting with B insted of type A. Use Type A insted of type B and conversely. Everything else remains the same. Thus the structure is different from case 1 but graph is invariant under E-cordiality.

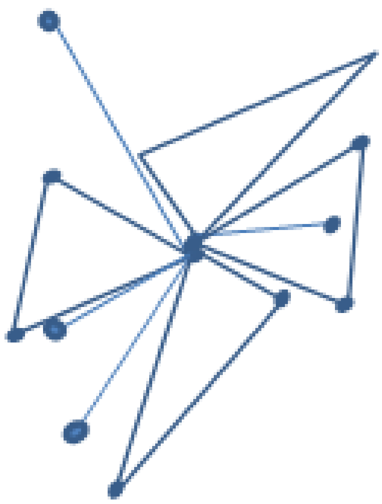

Fig 6: Case 2 G-unlabeled structure- not isomorphic to G in case 1.But E-cordial.

Case 3: We use 2-degree vertex as common point in G. In type A and Type B we use vertex x as a common point in forming G.Since f(x) and f(u) are same in both types we follow the labeling scheme as given in table 2 above. Rest of the results are same. The structure is not isomorphic to that in case 1 or case 2.But it is E- cordial.

Thus G= $(FL(C_3))^{(k)}$ i.e. One point union of k- copies of $FL(C_3)$ is E-cordial though it has 3 different structures.as explained in figures.(except for 3k≡1(mod 4))

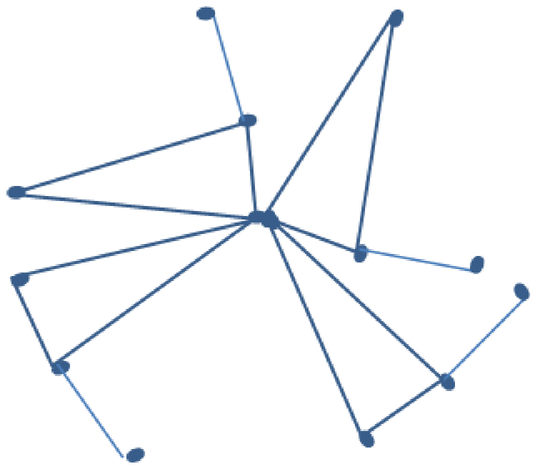

Fig 7: Case 3 G-unlabeled structure- not isomorphic to G in case 1and case 2.But E-cordial.

4.5. Theorem: G = $(FL(C_4))^{(k)}$ i.e. One point union of k-copies of $FL(C_4)$ is E-cordial.

Proof: The union point is taken on pendent vertex of $FL(C_4)$. The i^{th} copy of $FL(C_4)$ is defined as (u $e_{i1}u_{i2}$ e_{i2} u_{i3} e_{i3} u_{i4} e_{i4} u_{i5} e_{i5} u_{i2}); i =1, 2, k. We use two type of labeling namely type A and Type B to design G. Fig 3 gives details. The pendent vertex u is the common vertex of all copies of $FL(C_4)$ in G = $(FL(C_4))^{(k)}$.

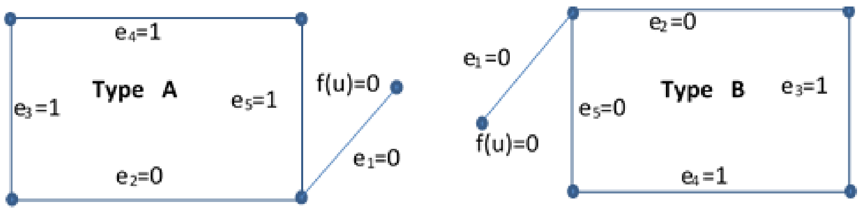

Fig 8: Type A and type B with Labels. Note in both types, vertex distribution is same. But edge distribution is different.

The copies of $FL(C_4)$ in $(FL(C_4))^{(k)}$ are numbered as 1, 2, k. The copies with number 1, 3, 5. are labeled with Type A label and the other copies with number 2, 4, 6, are labeled with Type B. The number distribution is $v_f(0)=2k+1$ and $v_f(1)=2k$ for given k.

$e_f(0)=e_f(1)=5x$ for k = 2x and for k=2x-1 $e_f(0)=5x+3$, $e_f(1)=5x+2$. The labeling is E-cordial.

5. Conclusions

There are different structures possible on $(FL(C_3))^{(k)}$. We have shown all possible 3 structuresand have obtained their E-cordial labeling. Thus the $(FL(C_3))^{(k)}$ is invariant under E-cordial labeling. We are sure that the same thing will be followed for $(FL(C_n))^{(k)}$ for given n. This will attract future attention.

6. References

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